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# Quarterly Progress Report

*Division 5*

# Information Processing

15 June 1959

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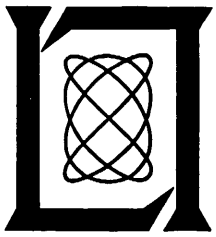
Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

*Lexington, Massachusetts*



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# INTRODUCTION

The introduction to the Division 5 Quarterly Progress Report dated 15 March 1959 discussed in a general way the programmatic goals of the newly formed Division in the diverse areas of mathematical analysis, digital computer component and application development, and man-machine relationships. The following are some of the highlights of the Division effort for the past quarter.

The TX-2 computer, a high-speed, large-memory, flexible machine with unique input-output features has begun to see some experimental use. Certain pieces of input-output equipment necessary for the nonroutine application of this versatile machine have been put into operation. For example, an analog-to-digital converter and a high-speed xerographic printer are now available for use, and a random number generator using a piece of radioactive material as a pulse source has been designed and is being tested. Graphical information connected with character recognition work is being successfully scanned and stored, and some machine time has been used for work on the speech recognition program.

A magnetic thin-film memory has been operating successfully in the Laboratory and will be tested further as a functional element of TX-2. Installation in TX-2 will be completed early in the next quarter. There are indications that thin magnetic films may be used in a rapid-access, high-density, fixed-storage device, and it is proposed that a modest fixed memory of this type be constructed to demonstrate feasibility. The mechanism of partial switching in ferrite cores has been investigated by means of wires inserted in "sense holes" drilled at various radial distances in the test core, and some preliminary work has been done on a nondestructive read-out technique in ferrite core memories. A study has been initiated on the potential of thin films as parametron elements. To this end, the necessary microwave equipment has been assembled and an initial investigation of the permeability of magnetic films at microwave frequencies has begun.

Some preliminary results have been obtained in multiple-station queuing theory, a mathematical area with wide application to military problems.

Computer programming has begun on the solution of some of the more subtle problems connected with the operational use of MAUDE (Morse Automatic Decoder), particularly those associated with the RF signal, i.e., fading, short- and long-term frequency drift and interference.

Writing computer programs for complex, nonnumerical processes is difficult. Some examples of such processes are answering questions posed in English, pattern recognition and theorem processing. Newell, Simon, Shaw and others have designed a computer language (IPL) for programming such complex processes. This language has been coded for the IBM 709 computer, to be used as a tool in the study of nonnumerical symbol manipulation processes.

W.B. Davenport, Jr.
Head, Division 5

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Associate Head, Division 5

19 June 1959

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# PUBLISHED REPORTS OF AUTHORS FROM DIVISION 5

2 March through 14 June 1959

## Technical Report

| TR No. | | | | Classification Report | Classification Title |
|---|---|---|---|---|---|
| 199 | Transistor Life Experience: 1954 – 1958 | D.J. Eckl<br>P.A. Fergus<br>R.L. Burke | 26 Mar. 1959 | U | U |

## Journal Articles

| JA No. | | | |
|---|---|---|---|
| 683 | The Effect of Signal-Rate on Performance in Visual Monitoring | H.M. Jenkins* | Am. J. Psychol. 71, 647 (1958) |
| 1111 | Anisotropy in Permalloy Films | D.O. Smith | J. Appl. Phys. 30S, 264S (1959) |
| 1112 | Properties of Permalloy Films Having a Magneto-Elastic Easy Axis Normal to the Film | E.E. Huber, Jr.<br>D.O. Smith | J. Appl. Phys. 30S, 267S (1959) |
| 1113 | Magnetic Properties of Nickel-Iron Ferrite | N. Menyuk<br>K. Dwight | J. Appl. Phys. 30S, 36S (1959) |
| 1115 | Internal Structure of Cross-Tie Walls in Thin Permalloy Films Through High Resolution Bitter Techniques | R.M. Moon | J. Appl. Phys. 30S, 82S (1959) |
| 1209 | Transistorized Generator for Pulse Circuit Design | L. Neumann | Electronics 32, 47 (1959) |

* * * * *

## UNPUBLISHED REPORTS

### Journal Articles

| JA No. | | | |
|---|---|---|---|
| 1241 | Measurement of Switching Transistor Parameters | R.C. Johnston<br>R.L. Burke | Submitted to Semiconductor Products Mag. |
| 1284 | Noncomputational Uses of Digital Computers | B.F. Green, Jr. | Submitted to Behavioral Sciences |
| 1288 | Thin Magnetic Films for Digital Computer Memories | D.O. Smith | Submitted to Electronics |
| 1290 | A Suggestion Concerning Magnetic Interactions in Spinels | D.G. Wickham<br>J.B. Goodenough | Submitted to Phys. Rev. Letters |

* Consultant.

## Meeting Papers

Titles of Meeting Papers are listed here for information only. No copies are available for distribution.

| JA No. | | | |
|---|---|---|---|
| 1262 | An Application of the Matrix Riccati Equation of the Two-Body Problem | J.J. Levin | Conference on Orbit and Space Trajectory Determination, NASA, Washington, D.C., 12-13 March 1959 |
| 1264 | Results Obtained from a Vowel Recognition Computer Program | J.W. Forgie<br>C.D. Forgie | Acoustical Society of America, Ottawa, 14-16 May 1959 |
| 1270 | The Rate of Learning of Serial Digits | N.C. Waugh | American Psychological Association, Cincinnati, 3-9 September 1959 |
| 1271 | On the Statistical Analysis of Discrimination Matrices | J.E.K. Smith | American Psychological Association, Cincinnati, 3-9 September 1959 |
| 1294 | Work in Progress on the Automatic Segmentation of Speech into Syllable-Like Units | A.N. Stowe | Fourth Conference on Linguistics, Hunter College High School, New York, 9 May 1959 |
| 1296 | Some Magnetic Components Under Development at Lincoln Laboratory | J.B. Goodenough | Working Group on Electronic Materials, AFCRC, 28 April 1959 |
| 1301 | Transistor Circuit Techniques for a Core Memory with 500-mμsec Cycle Time | V.J. Sferrino | IRE-WESCON, San Francisco, 18-21 August 1959 |
| 1305 | A Computer Memory Using Magnetic Films | J. Raffel<br>D.O. Smith | International Conference on Information Processing, Paris, 13-23 June 1959 |
| 1314 | Self-Organizing Models for Learned Perception | B.G. Farley | Information Systems Branch, ONR, Illinois Institute of Technology, 5-6 May 1959 |
| 1317 | An Error Correcting Encoder and Decoder for Phone Line Data | K.E. Perry | IRE-WESCON, San Francisco, 18-21 August 1959 |
| 1331 | A Study of Iron, Nickel, Cobalt Nitrides | R.J. Arnott<br>A. Wold | American Crystallographic Association, Ithaca, 19-24 July 1959 |

# DIGITAL COMPUTERS
# GROUP 51

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# DIGITAL COMPUTERS
# GROUP 51

## I. SUMMARY

The TX-2 computer is now officially in operation. Many of its major elements have been operating for some time (most of the main memory operated with TX-0 three years ago), and most of the basic ensemble of elements have been running together for many months. Now, however, the machine's operating time is no longer exclusively used in testing and evaluation; a small but increasing fraction is being used in research applications. Graphical information, such as pictures and symbols, is being successfully scanned, stored, processed, and redisplayed by the computer in connection with character-recognition experiments; some time has been spent debugging the programs of the speech-recognition project; and a small start has been made on building programs for neuron-net research. All of this, plus checking and running of utility, test, and demonstration programs occupied over half of TX-2's operating time by the end of the quarter; this fraction will gradually increase, as will the total amount of operating time, even though equipment (both central and peripheral) continues to be added to the complex. Checking margins, debugging new additions, and educating and training personnel (users as well as operators) occupied the remainder of the machine's operating time.

Further progress in plated interconnections includes fabrication of the first example of a multilayer piece of "back panel" wiring.

The first magnetic-film memory has been in successful test operation; it will be installed in TX-2 for further evaluation during July. In the meantime, film uniformity continues to receive heavy emphasis as the major obstacle to the early realization of large memories.

W. A. Clark
B. M. Gurley
W. N. Papian

## II. COMPUTER DEVELOPMENT

### A. TX-2

#### 1. Central Machine

Major emphasis has been directed toward increasing the reliability of the system. Margin checking has begun to be useful in locating sources of potential trouble, and the reliability of the computer has increased rapidly with very few failures reported during applications time.

Some time ago the lugs on the etched cards in the plug-in units were found to be intermittently separating from the printed lines; as a result a program of resoldering all boards was initiated, and lugs were replaced by eyelets in all new construction. The resoldering program was satisfactorily completed by the end of the quarter.

The capacity and regulation of many of TX-2's original power supplies are becoming inadequate to handle the increasing load. Specifications for new ones have been written and submitted to various vendors.

Experience gained operating and testing TX-2 has indicated that some modes of operating input-output devices were not useful enough to justify the additional equipment required in the control of those modes. Consequently, they will be eliminated. In addition, the "transfer data to output devices" instruction will be modified so that an alarm will no longer occur if the device

is not ready; instead the computer will hold the data, perform some other operation, and transfer the data at a later time when the output device is available. The modifications will be made next month.

J. W. Forgie
K. H. Konkle
C. S. Lin
C. A. Norman

2. Memories

Two significant faults were found and rectified in the 65,536-word memory. First, a short between a sense winding and digit winding was discovered in one of the planes; one extra connecting shim had been mistakenly inserted during construction and remained undetected until this time. Second, the gate circuit where the sense amplifier output is strobed was found to be operating marginally because of an abnormal fall time; the circuit was successfully redesigned for shorter fall time.

Equipment for the 4096-word transistor-driven magnetic-core memory is now arriving from various vendors and the installation should be completed this summer.

A new, small unit of fixed storage to augment the plugboard and toggle-switch storage in TX-2 has been designed. Photorectifiers developed by Group 26 are used to read the 24 words from a single IBM punched card. Evaluation of the device will be completed when the card-holding mechanism is received from the manufacturer.

The control logic and driving circuits of the magnetic-film memory for configuration control are being debugged and tested; the memory will be installed in the computer during the next quarter.

J. L. Mitchell
L. Neumann

3. Input-Output

The typewriter and keyboard for the prototype Lincoln Writer have not arrived from the manufacturer and therefore its completion will be delayed until early summer. Parts for five more Lincoln Writers are ordered and being constructed; the Writers will be available to programmers soon after the evaluation of the prototype is completed.

The Datrac analog-to-digital converter has been installed and checked out in TX-2, and is now available to programmers.

A random-number generator has been designed and is now being tested. It uses a piece of radioactive material as a source of pulses which occur at random times. After the pulses have been detected and amplified with a photomultiplier, they are gated at regular time intervals into a flip-flop. In order to eliminate biased results due to long-term drift of voltages and components, two successive settings of the flip-flop are compared, and only if they are different is the output accepted as random. On the average the system will generate a 9-bit random number every 50 μsec.

The spare selenium drums for the high-speed Xerographic printer were replated by the manufacturer and one was installed in the printer, replacing the unit with the scratched and deteriorated surface. Good printed copy is now available to machine users. The printer will be complete when the charactron tube with the 88 Lincoln Writer characters is received from Stromberg-Carlson and installed.

Some new equipment has been added to the TX-2 display which allows it to perform as a program-directed spot scanner for experiments in picture and character reading. The system consists of a photomultiplier detector which, using optical focusing, looks at the entire face of the display cathode-ray tube through the film to be read. The output of the photomultiplier is fed back to the machine via the electronics ordinarily used by the light pen, and the picture is stored in TX-2's main memory. Figure 51-1 is such a picture as redisplayed by the computer during one of the first experiments with this technique. A finished system to replace the breadboard prototype is being designed and a report describing the scanner and experimental results is being written.

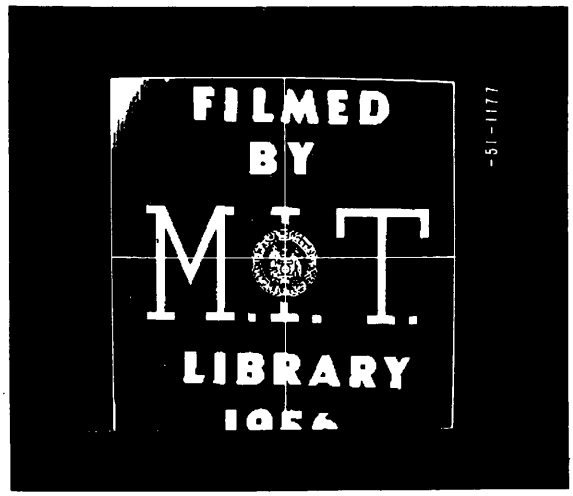



Fig. 51-1. Microfilm leader, as redisplayed, using a 256 × 256 raster and one pass.

A. W. Case
R. H. Gould
L. Neumann
H. P. Peterson, Jr.
R. E. Savell

4. Magnetic Tape System

The major part of the electronics for the magnetic tape system has been received from the shop and is being assembled and inspected; however, the tape transports have not yet arrived from the vendors.

One of the problems has been false timing signals due to the large noise signal that is inductively coupled into the timing head when information is being written elsewhere on the tape. Two solutions appear promising: integration of the timing track signals, and disabling the timing amplifier during write activity.

J. E. Laynor
T. C. Stockebrand

5. Test and Utility Programs

Test programs for each part of the machine have been incorporated into one complete routine that tests each part many times in turn, printing out any errors that may have occurred.

Both low- and high-speed printout routines are available, and a flexible typewriter input routine for program monitoring, debugging and changing is nearly ready.

A flexible symbolic-address assembly program is well on the way to completion. In addition to the ability to express computer words and numbers by any desired symbology, the programmer has a choice of his own "format," and choice of "macro-instructions" (many-word instructions). Furthermore, he will be able to "converse" with the computer at the console in his own symbolic language for change and debugging.

Patricia A. Fergus
J. T. Gilmore, Jr.
R. P. Mayer
H. P. Peterson, Jr.
A. Vanderburgh

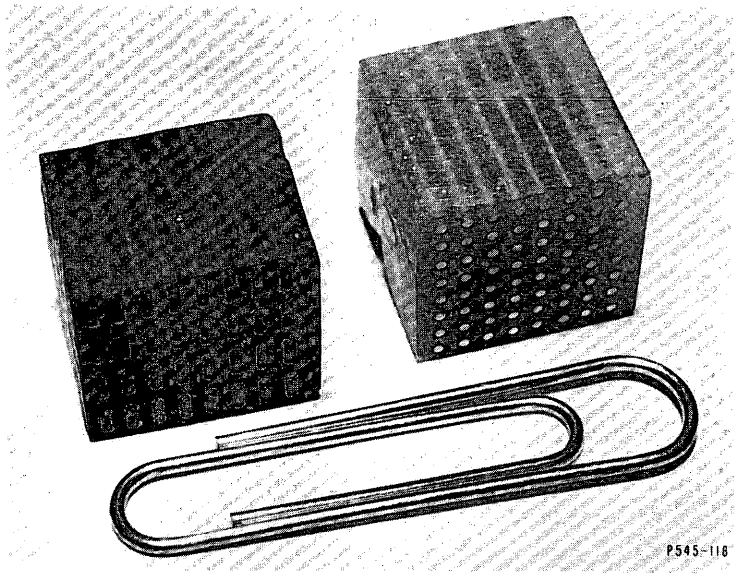



Fig. 51-2. Test sample of 64 encapsulated wires joined in series by 128 plated connections and deposited jumpers (left). Before engraving and copper deposition (right).

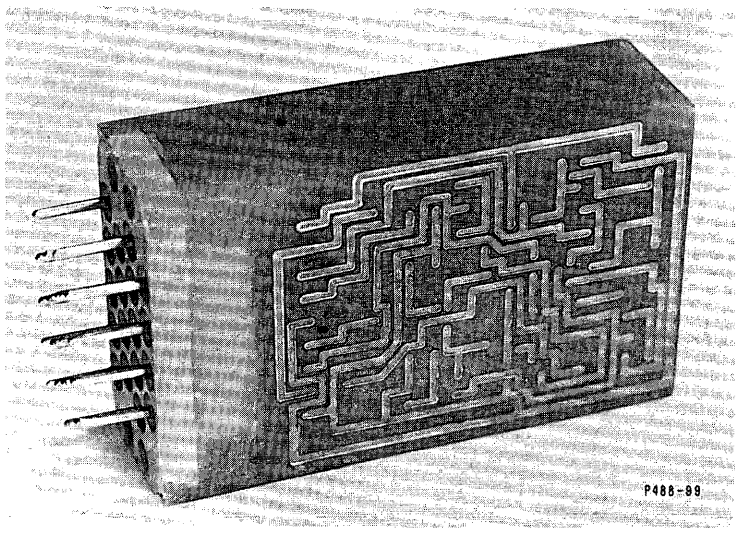



Fig. 51-3. Encapsulated high-speed computer flip-flop with deposited wiring and plated connections.

## B. Fabrication Techniques

### 1. Plated Component Connections

Evaluation of plated, or deposited, component connections has continued with the test unit shown in Fig. 51-2. Sixty-four copper wires (AWG No. 22, 0.025 inch diameter, spaced 0.015 inch deep) are engraved on opposite faces of the cube so that subsequent metalizing forms 128 plated connections and jumpers that connect all 64 wires in series. The average combined resistance of a plated connection and jumper is one milliohm.

The encapsulated flip-flop of Fig. 51-3 has components identical to those used in the standard TX-2 computer plug-in unit. Conductors, deposited in grooves engraved in the plastic (both sides), make plated connections to the component leads faced off during engraving. The package contains 59 components and 149 plated connections. Its size, compared with the TX-2 counterpart, can be seen in Fig. 51-4; a 50 per cent volume reduction would be about the maximum attainable with this packaging approach and standard components. A unit of this type was placed in operation in the TX-2 computer on 1 May 1959.

Figures 51-5 through 51-7 show successive deposition stages for three layers of "back-panel" wiring with plated connections to six 18-pin Winchester plugs. The encapsulated plugs and wiring, in conjunction with three TX-2-type high-speed flip-flops and two P-5 inverter packages, comprise a binary counter for demonstration and reliability testing.

### 2. Core-Memory Plane Frames

A departure from conventional plane-frame design (Fig. 51-8) is shown in Figs. 51-9 and 10. The upper and lower parts of the new frame (executed for us by Corning Glass Works) are photoetched from photoform glass, assembled and heat treated to become a fused photoceram unit. The etched-ceramic terminals are then metalized so memory-core drive wires and frame-to-frame jumper wires can be soldered to them. The advantages of this design are a relatively low initial setup (artwork) cost, regardless of shape or size of frame, plus the reduced assembly time and expense of handling individual terminals.

### 3. Microprobes

An attempt is being made to produce a multiple-contact probe for sensing of bioelectric signals. The ultimate goal is 100 contacts/$mm^2$ but present efforts are directed toward a 25-contact probe with a 0.0005-inch diameter tungsten wire spaced on 0.005-inch centers in a square array. The work is divided into three principal steps: (a) positioning the probe wires according to prescribed dimensions; (b) fanning the output ends of the probe wires and attaching them to terminals or a plug and encapsulating the entire assembly in a suitable plastic; (c) cutting and polishing the end of the assembly at an elevated temperature, so that the plastic, when cooled to the operating temperature, will shrink relative to the lubricated wires. The wires then project beyond the plastic for the desired distance (about 0.0005 inch).

E. A. Guditz

# III. MAGNETIC FILM ENGINEERING

## A. Magnetic Film Memory

### 1. TX-2 Configuration Memory

A section of the TX-2 computer frame has been wired to accept the magnetic-film "configuration" memory, its driving and sensing packages, parity circuits, delay lines and other

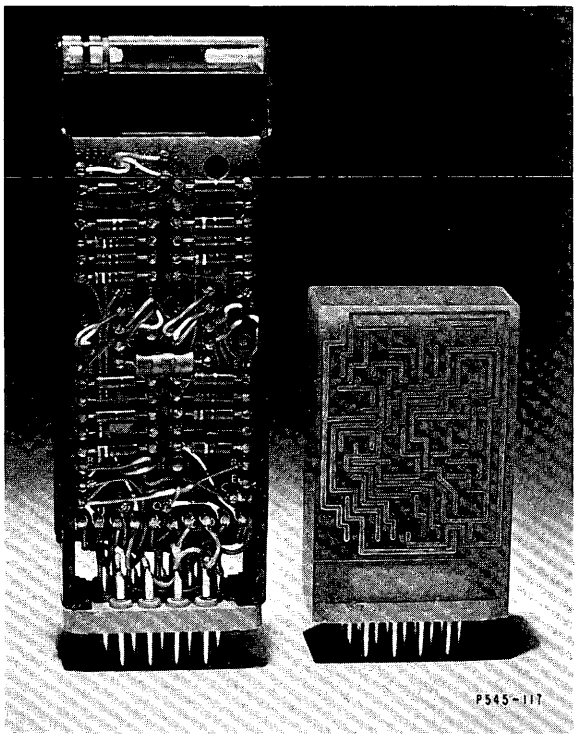



Fig. 51-4. Size comparison of standard pluggable unit (left), and encapsulated version with identical components (right).

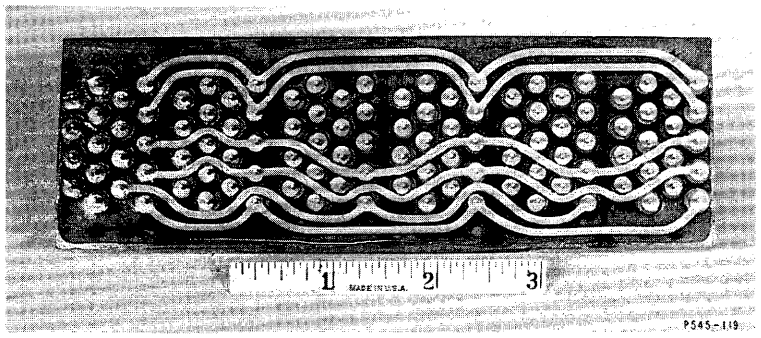



Fig. 51-5. First layer of deposited wiring and plated connections to receptacle extension pins.

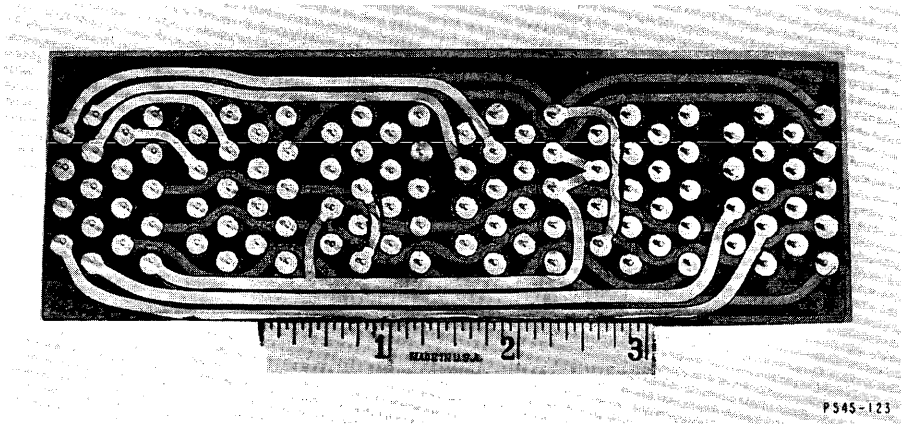



Fig. 51-6. Second layer of deposited interplug wiring and plated connections.

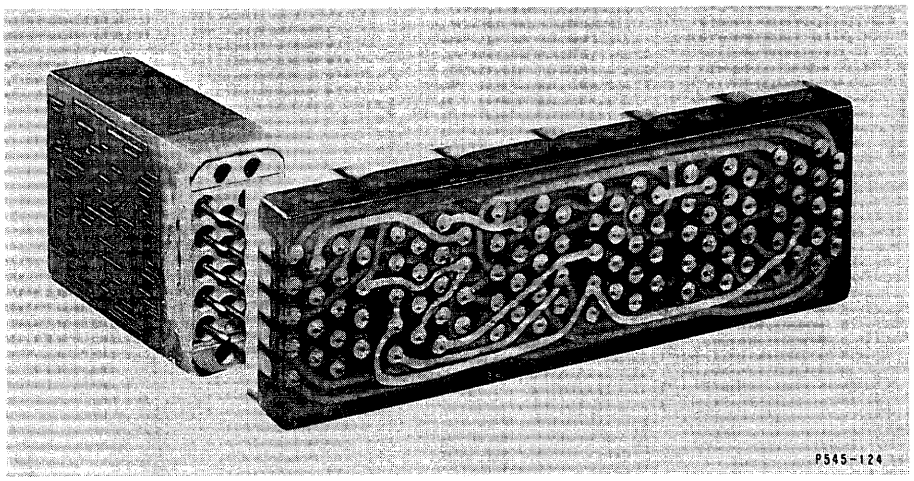



Fig. 51-7. Completed 3-layer deposited back panel wiring of 3-stage binary counter.

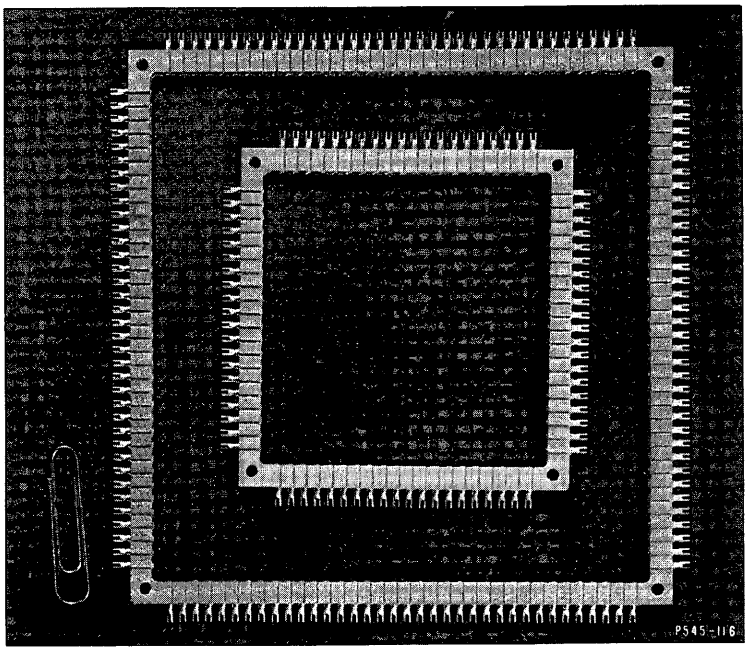



Fig. 51-8. Conventional memory-plane frame construction employing machined phenolic material and swaged terminals.

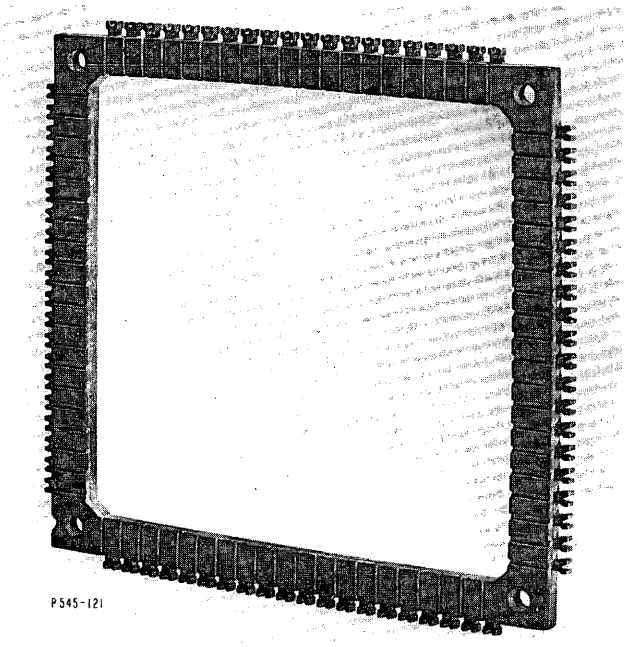



Fig. 51-9. Photoetched ceramic memory-plane frame with metalized terminals.

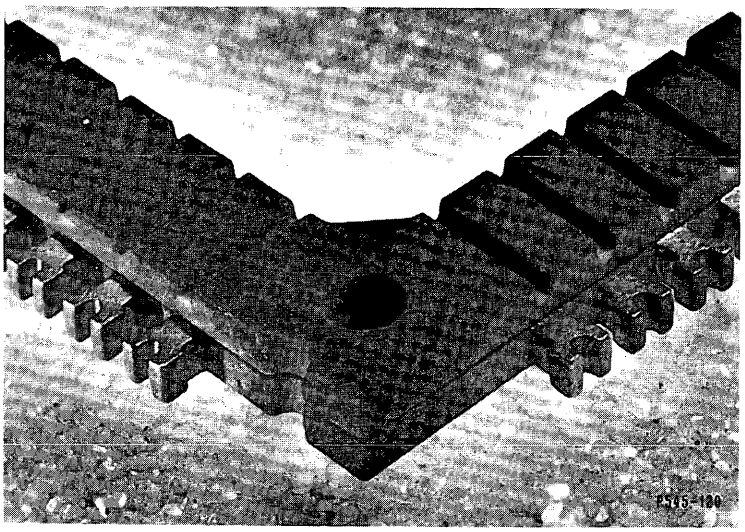


Fig. 51-10. Closeup of ceramic memory frame showing fused 2-layer construction and detail of metalized etched-ceramic terminals.

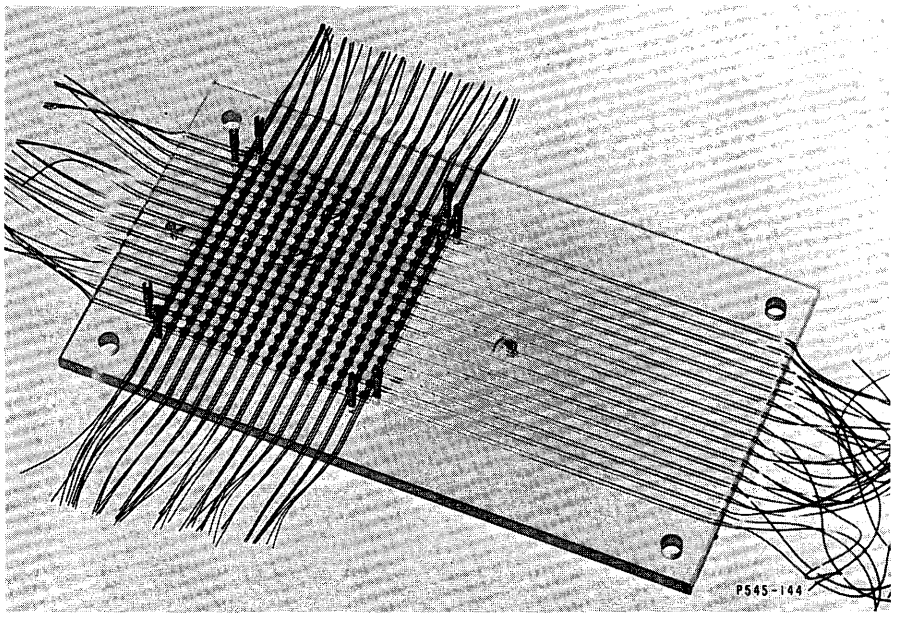



Fig. 51-11. Wiring mat.

associated logical packages. The memory has 32 ten-bit words and will operate in the computer with a total read-write time of 0.8 μsec, of which approximately 0.4 μsec will be for the actual read-write cycle, 0.2 μsec for parity checking, and 0.2 μsec for timing margin.

Primary checkout, using toggle switches for simulation of TX-2 input information, has shown that all ten digits are functioning satisfactorily. Upon the installation of an improved film holder and films, the few parity alarms now attributable to the memory proper should be eliminated and the memory will be installed in TX-2.

T. O. Herndon

2. Memory-Plane Wiring Mat and Holder

In order to make a wiring mat that is stable, flat and transparent, glass was chosen as the material for the top-half section, and fiberglass (painted white for visual contrast) for the bottom-half section. The drive and sense wires are laid down, using a modified epoxy-polyamide formulation as a bonding agent after the wires have been positioned by means of a winding jig. The film plane is sandwiched between the two sections and is held by two positioning pins at each corner (Fig. 51-11).

3. Plane Handler

The first prototype of a new magnetic-film plane handler for the array tester has been built. This is now being redesigned and provision made for motorizing the x and y movements.

R. C. Zopatti

## B. Evaporation of Permalloy Films

1. Angular Dispersion of Easy Axis

Two methods for measuring the angular dispersion of a film's easy axis have been investigated. In both methods a saturating sinusoidal field is applied perpendicular to the easy axis. (The longitudinal flux is then observed to be zero.) In the first method the quartile angle of the

distribution is obtained by rotating the film until the peak flux is half the saturation value. In method number two the same thing is accomplished by applying and varying a DC field in the direction of the easy axis. The two methods yielded different results as detailed in Group Report 51-2 (not generally available). The investigation is continuing.

2. Optical Anisotropy

Films made at a 45° angle of incidence to the vapor beam have been shown to be somewhat optically anisotropic in the transmission of white light, the effective density being less in the direction in which the vapor was deposited.

3. Uniformity Problem

Twenty-five per cent of one hundred twenty-two 900 Å, 16 × 16 arrays of $\frac{1}{16}$-inch diameter spots made on a moving substrate passed looper specifications of rotational coercive force ($H_k$) less than 4 oe, and limits for the threshold for wall switching ($H_{cm}$) given by $\frac{1}{2} \leqslant H_{cm}/H_k \leqslant 1$. Nearly all failures were due to $H_{cm}/H_k < \frac{1}{2}$. These figures are all average measurements over the entire plane. Pulse testing of those arrays that passed average looper specifications still showed spot-to-spot uniformity to be a problem. The addition of an ethyl alcohol wash preceding the cleaning procedure reported in the Division 5 Quarterly Progress Report for 15 March 1959 is being tried to obtain cleaner glass and thus higher wall coercive forces.

4. Production Unit

The production unit with a capacity of forty-eight 16 × 16 arrays has been designed and drawings are being prepared for construction. It will be installed during the next quarter.

5. Moving Substrate

It has been observed that there is a noticeable difference between films made on a moving substrate and those made on a stationary one. This seems quite remarkable and the effect is being examined in more detail. Table 51-I shows the results of stationary vs moving substrate evaporations. In each of two runs, eight $\frac{1}{2}$-inch spots were evaporated on a moving substrate and then eight more on a stationary substrate. The average deviations and average values of both $H_k$ and $H_c$ are much higher on the moving runs.

TABLE 51-I

| Run No. | | Thickness Å | $H_c$ (wall coercive force, oe) | | | | $H_k$ (rotational coercive force, oe) | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Average | Min. | Max. | Average Deviation from Mean | Average | Min. | Max. | Average Deviation from Mean |
| 312 | Stationary | 600 | 2.2 | 1.9 | 2.4 | 0.2 | 2.7 | 2.3 | 3.2 | 0.3 |
| | Moving | 800 | 4.6 | 2.0 | 8.0 | 1.6 | 3.2 | 2.0 | 5.0 | 1.1 |
| 313 | Stationary | 1800 | 1.2 | 1.0 | 1.6 | 0.2 | 2.6 | 2.3 | 3.0 | 0.2 |
| | Moving | 1400 | 4.3 | 1.2 | 6.6 | 2.1 | 3.5 | 1.8 | 5.6 | 1.7 |

T. S. Crowther
J. I. Raffel

## C. Testing and Instrumentation

Some of the problems associated with phase shift in the pickup circuit of the low-frequency hysteresis looper have been eliminated by modification of the integrator. The new "integrator" network is designed so that the zeros of its transfer voltage ratio cancel the poles of the pickup circuit transfer function. As a result, the damping resistor is no longer required to eliminate ringing. The modification reduces frequency dependence, and makes possible a more faithful display of film hysteresis loops. A report is being prepared which describes the circuitry in more detail.

A second looper has been constructed and will be in operation shortly.

D. D. Strassberg

## D. Circuits

Work continues on the diode matrix switch for word selection. A sketch of a 16-position version is shown in Fig. 51-12. A 128-position pilot model has been constructed and is now being evaluated with encouraging results. Using modified configuration memory drivers, 250-ma current pulses of 60- to 80-mμsec duration may be directed to any one of 128 two-turn loops that provide the transverse switching field for the film elements. A ground plane tightly coupled to the matrix, which has planar geometry, succeeded in shortening current rise times and should become increasingly effective as memory size increases. Measurements of the noise coupled to the sense line from the matrix at sensing time are being made. Modifications in the matrix layout to reduce the noise will be incorporated in the next model. The application of this selection scheme to a larger memory (1000 words) should be straightforward, although the noise problem will be more severe. In this connection the common-mode rejection of the sense amplifier is being measured in detail, and there are indications that it is much poorer than was anticipated. Further work is being done to determine changes necessary to improve the rejection.

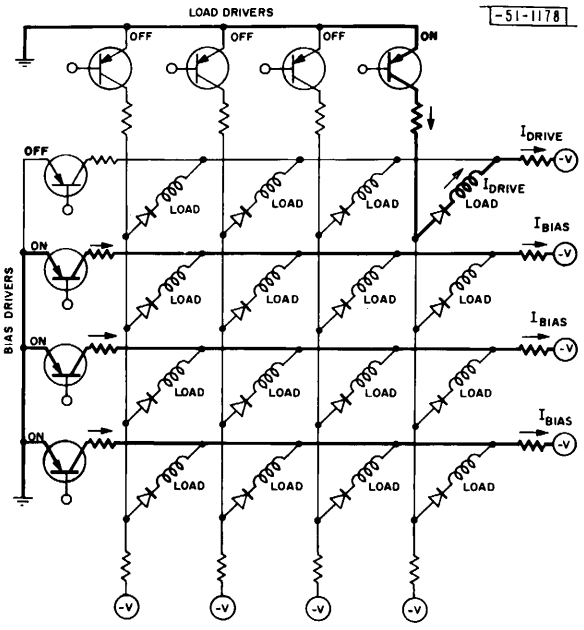



Fig. 51-12. A 4 × 4 diode matrix.

H. Blatt

## E. Electrodeposition of Magnetic Films

The subcontract with General Electric has produced a masking technique which appears satisfactory. After plating, the areas to be preserved are coated by standard photographic techniques and the unwanted permalloy is etched away with $FeCl_3$.

Data from General Electric also indicate that $H_c$ is proportional to $\delta^{-4/3}$ ($\delta \triangleq$ thickness) but that $H_k$ does not change appreciably with thickness. It is possible to control $H_k$ by adjusting

film composition. Furthermore, as reported earlier, $H_c$ may be adjusted in films of given thickness by changing the voltage at which the gold substrates are sputtered. Thus, by selecting appropriate sputtering voltage, bath composition and plating time, it should be possible to produce films to meet reasonable specifications on $\delta$, $H_c$ and $H_k$.

A new vacuum unit for sputtering gold substrates has been installed.

D. D. Strassberg

## F. Fixed Storage Using Films

A Master's thesis which proposes a fixed storage system with rapid access and high storage density has just been completed and will be available as Group Report 51-5.*

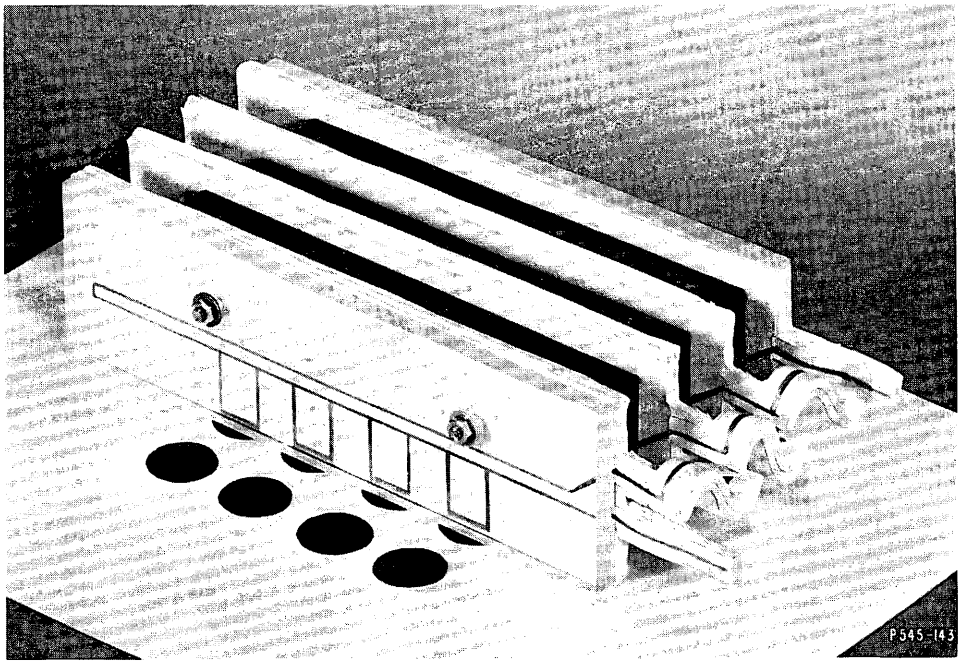
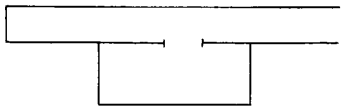


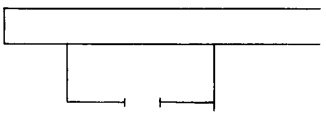

Fig. 51-13. Model of storage sense line.

Figure 51-13 shows a paper mockup of the storage sense lines and magnetic film elements for a 16-word memory. There are four rows of four spots (plus an unused row of spots shown at the left for illustration). Each film element is identified with one word. The storage sense line is a standard pattern etched from some thin, copper-coated material such as Mylar. As shown in the photograph the sense lines are assembled so that several, in this case 40, are coupled to each element. Information is stored in the sense lines before assembly by opening the upper lead to insure the presence of flux coupling to the spot (ONE), or the lower lead for absence of coupling (ZERO), as indicated below. It should be possible to mechanize selective perforation of a standard etch-wire pattern.

*A.H. Anderson, "Magnetic Film Random-Access Permanent Memory," Group Report 51-5, Lincoln Laboratory, M.I.T. (5 May 1949) (not generally available).



ONE



ZERO

The desired word is read by switching the appropriate spot with coincident currents. Experiments indicate that it may be possible to use films that exceed the critical thickness for single domain remanence. Films 6000 Å thick have been used and have shown a three-domain pattern. A memory of about 256 words with 30 to 40 digits is being considered as a first goal.

A. H. Anderson
J. I. Raffel

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# DATA PROCESSING
# GROUP 52

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BAKER, R. H., *Assistant Leader*
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# DATA PROCESSING
# GROUP 52

## I. INTRODUCTION

That part of the Group 52 effort devoted to the AICBM program has been reported in the AICBM quarterly progress report, including the CG 24 computer modifications, the prediction programs and methods, and the infrared research.

The remaining part of the group program is concerned with the development of new computer techniques for data-processing systems, with the construction of a data-transmission system utilizing coding techniques for the detection and correction of multiple errors that may occur in transmissions, and with theoretical investigations of various encoding systems.

A new technique that has been investigated during the past quarter as a possible method of performing a nondestructive core memory read out looks very encouraging. For normal computer uses, the possibility of nondestructive memory read out does not appear to be very significant, but for real-time data-processing applications the availability of a program storage system that is "fixed," but can also be changed easily should offer a significant improvement in reliability.

E. W. Bivans

## II. MEMORY DEVELOPMENT

### A. Magnetics Research

The study of the partial switching mode of operating ferrite cores has been continued in the Group 52 magnetics program. From experimental data obtained on the effects of varying the amplitude and width of the current pulses, mathematical models have been proposed to describe the properties of the switching process such as switching time, the amount of flux switched and the switching waveform.

The experimental work has been conducted on cores that have small "sense holes" drilled within the main body of the toroidal form of the core so that the flux at various positions within the core can be measured during the time that the entire core is switching. It has been postulated that the switching process is greatly affected by the geometry of the core, and that the driving field intensity (H) decreases with distance (R) from the drive wire in an inverse R relationship. The amount of flux switched and the speed of the switching process at various locations within the core is therefore of prime interest. By measuring the amount of flux switched within the core due to very low drive current pulses of long duration it was found that the field intensity at the outer edge of the core was insufficient to switch the core. However, the inner region located closer to the drive wire switched almost completely. Tests on these cores with high drive current pulses were also made showing that the outer region switched slower than the inner region. A sketch of the two waveforms for a core of 0.030 i.d. and 0.050 o.d. is shown in Fig. 52-1. When the width of this high-current pulse is reduced, the core can be partially switched because the drive pulse is not of sufficient duration to allow completion of the switching.

Large cores of approximately $\frac{1}{4}$ inch o.d. with various i.d. were fabricated. These were investigated to study the effect of the i.d. on the current value at which switching is initiated. An interesting result of these tests is that the current at which flux begins to switch (noted as $I_o$ in Fig. 52-2) is reduced when the i.d. of the core is reduced but the location of the break in the switching characteristic curve remains almost constant. This result seems to indicate that the

lower portion of the switching characteristic curve occurs for low current values where the core is not being fully saturated and the break in the curve occurs at the point of complete saturation.

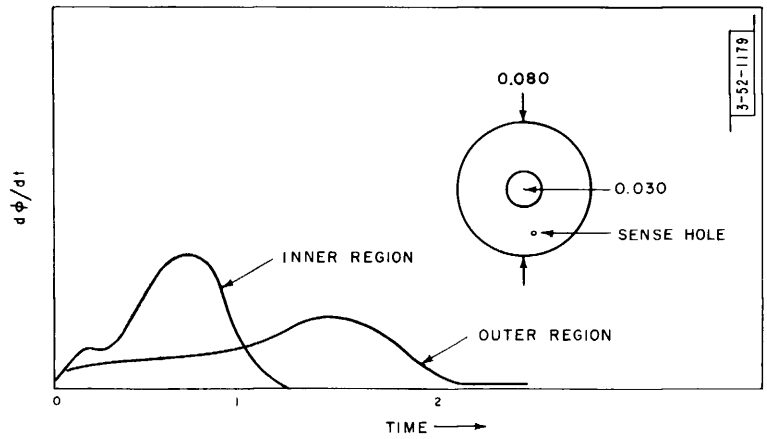


Fig. 52-1. Switching waveform for two regions in an experimental core.

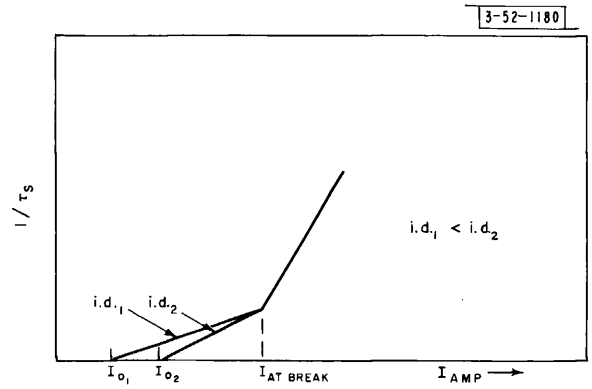


Fig. 52-2. Switching characteristics for changes in i.d. of core.

In an attempt to formulate the results in a consistent manner, mathematical models have been proposed. In one model the flux at any given point in the core is assumed to switch at a speed proportional to H at that point. Further, it is assumed that the component of flux in the direction of the applied field can be represented by a sine curve of $\phi$ vs t until it reaches saturation in the direction of the field. The equation has the form

$$\phi_{\text{at R}} = -\phi_s \cos K(H - H_o)\, t \qquad \text{for} \qquad t < \tau_s \quad , \tag{1}$$

where $H \propto I/R$, I is the driving current amplitude, R is the distance measured from the center of the core, $\phi_s$ is the saturation flux, and $\tau_s$ is the time required for the flux at R to align itself with the applied field. Both K and $H_o$ are obtained from experimental data for a particular material. From this equation, the rate of change of the flux within various portions of the core can be calculated, as well as the percentage of flux switched in the various regions at a given time. To obtain the total flux $\phi$, Eq. (1) must be integrated with respect to R over the region of the core under consideration. The results from approximate calculations of this integration were reasonable enough to warrant a more accurate solution of Eq. (1) as a function of R and t

by machine computation. Numerical integration programs have been written for CG 24 and a more accurate check of the model can now be made.

In addition to the work on studying partial switching phenomenon, tests have been made on some of the properties of the cores as used in the Group 47 memory. Cores chipped during assembly into the modules were tested to find to what extent the chips changed the partial switching characteristics. Very small chips had no noticeable effect on the switching waveform and only large chips that could be seen easily by eye produced a significant deterioration.

R. H. Tancrell

## B. High-Speed Memory for Group 47

The assembly of the final memory system has progressed smoothly. Power supplies were tested and installed, block diagrams and wiring sheets were initiated, and all of the core arrays were received and tested mechanically by the wiring shop. The sliding assembly which will house the core arrays has been installed along with the 1200 wires that connect to them, and a major portion of the timing circuitry has been installed and operated.

Although the exact requirements for the Group 47 system have not been established, it was decided to incorporate the facility of writing a given memory word back into an address in the same cycle in which it is read out. Figure 52-3 is a diagram of the memory transfer logic which controls the transfer of information into and out of the memory. The strobe gate enables only the read output from the sense amplifier to be transferred to the output register. The method of strobing described in the previous quarterly progress report was tried and abandoned when it was discovered the pulses coupled from the exciter to the sense winding were occasionally of sufficient amplitude to appear at the output of the sense amplifier. Seven bits of the small memory were connected to the transfer logic and all channels worked well.

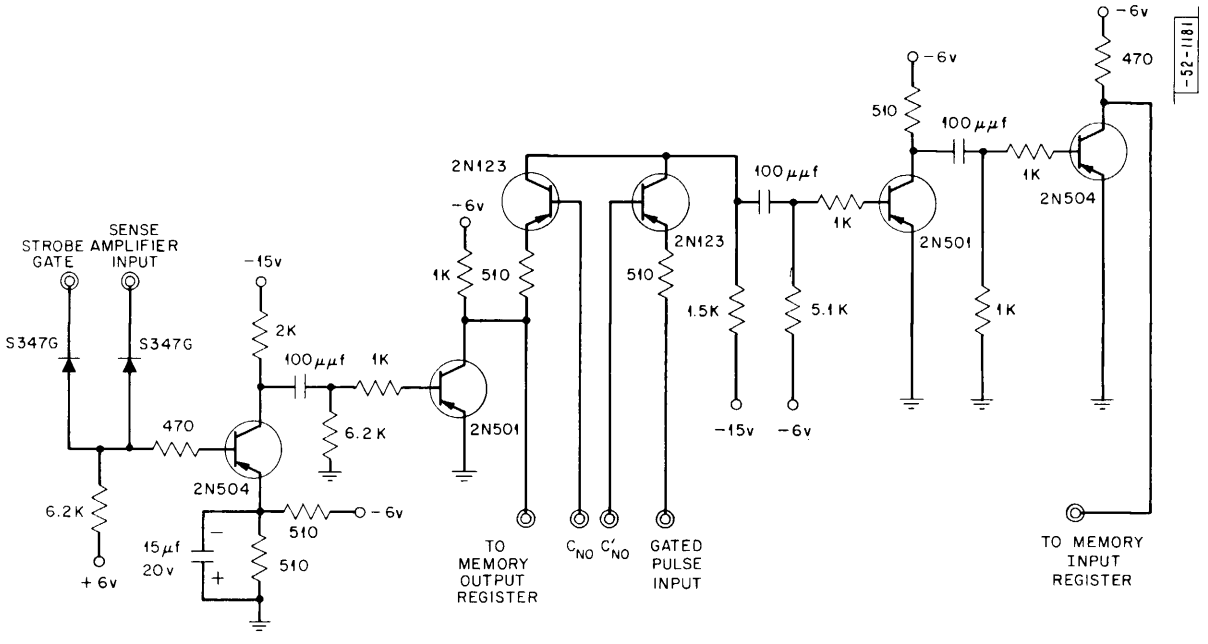


Fig. 52-3. Memory transfer logic.

One additional circuit was designed for use in the final memory system. The logic buffer (Fig. 52-4) is used to provide a logical output capable of accepting 45 ma of current when the logic

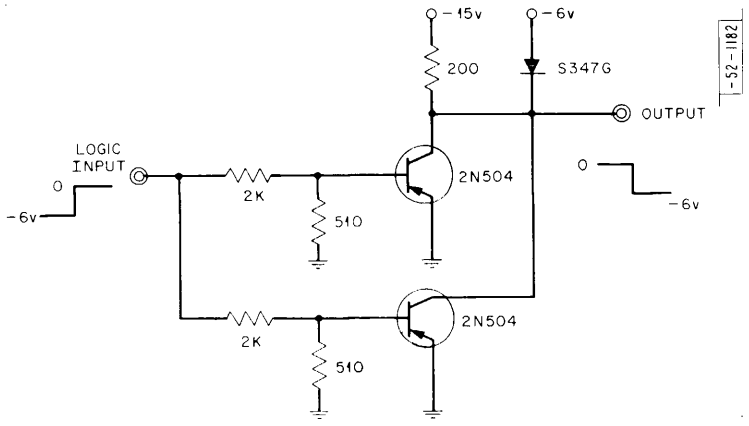


Fig. 52-4. High-speed logic buffer.

level is intended to be –6 volts. The input to the buffer is the complement of the desired level since the circuit inverts the level. These buffer units will be used following flip-flops that control a large number of logical inputs.

V. J. Sferrino

### C. A Nondestructive Read-Out Technique

In order to justify the assumption that the output voltage of a core being switched is directly related to the wall area changes rather than to the velocity of wall motion, the application of a small sinusoidal current has been applied during the switching interval.* An extension of this technique has provided some interesting results.

If the sinusoidal current is small in amplitude and below 5 Mcps in frequency, the experimental results are as would be predicted; i.e., the variation of output voltage is maximum at the peak of the normal output voltage, indicating maximum wall area at this point. If the sinusoidal current is very large, it opposes the normal drive current so that switching is very slow as shown in Fig. 52-5.

Now, if the sinusoidal current frequency is greater than 5 Mcps, a noticeable increase in the superimposed AC output signal ($V_{AC}$) occurs continuously (Fig. 52-6). Some further data are observed by partially switching either the read or write interval. Under these conditions, the AC output signal following a partial switching current is over three times as large as when the core is fully switched (Fig. 52-7).

The characteristics of this type of operation are best illustrated graphically as shown in Fig. 52-8. The curve indicates that the phenomenon is of practical value only if the sine wave drive frequency is greater than about 5 Mcps. A second important observation is that the high-frequency drive does not affect the regular switching process as does the low-frequency drive even at much higher current levels.

One obvious application of this technique is for a nondestructive read-out memory system. A possible mode of operation of either a memory or logic network using this technique is to allow a sine wave drive of 10 Mcps to be continuously applied to the cores. The AC output may then be

* J. J. Becker, J. Appl. Phys. 30, 387 (1959).

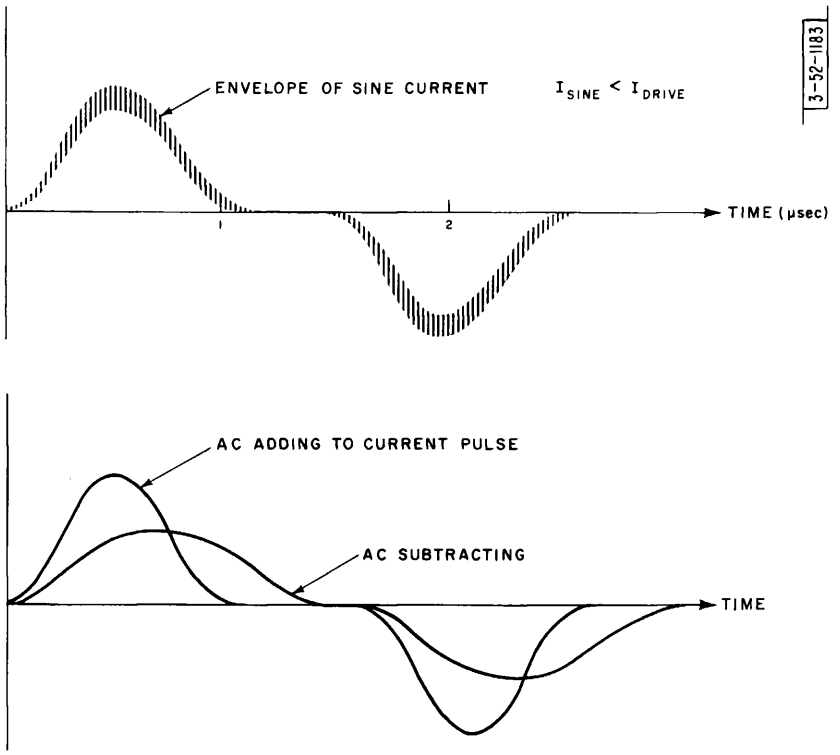


Fig. 52-5. Effect of low-frequency sine wave superimposed on core drive current.

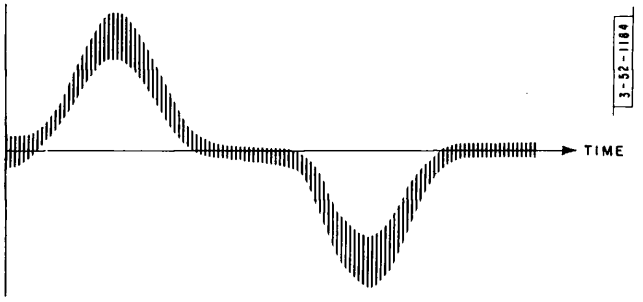


Fig. 52-6. Core output for a high-frequency sine wave superimposed on drive current.

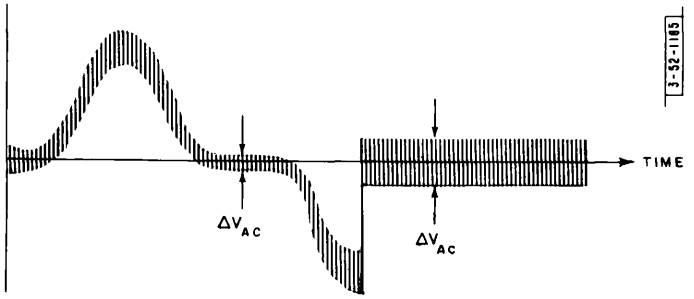


Fig. 52-7. Output signal for partially switched core with sine wave superimposed on drive current.

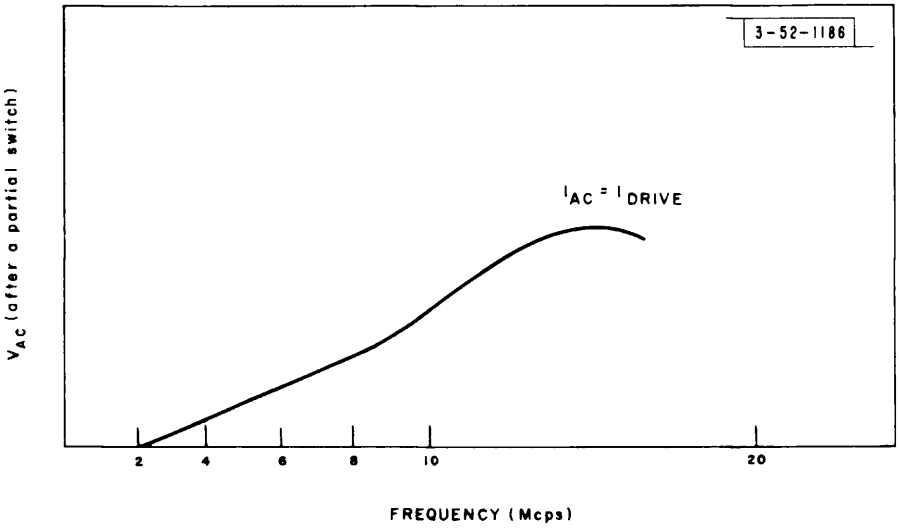


Fig. 52-8. $V_{AC}$ vs frequency for partially switched core.

rectified or used directly to indicate the state of the core, giving a continuous read out that may be interrogated at any time. AC power requirements are modest; the sine wave current level may be as low as 20 ma.

R. E. McMahon
E. W. Bivans
R. H. Baker

## III. DIGITAL DATA TRANSMISSION

### A. Coding Theory

The properties of certain error correcting codes are being studied by considering the expansion of step functions representing these codes in orthogonal functions.

Let $H^m$ be the class of all bounded step functions of the form

$$f(x) = f_i \quad , \quad x \in \left[\frac{i}{2^m} \quad , \quad \frac{i+1}{2^m}\right] \quad , \quad i = 0, 1, \ldots, 2^m - 1 \quad .$$

Let $K^m$ be the subset of $H^m$ in which $f_i = \pm 1$, where $i = 0, 1, \ldots, 2^m - 1$. Any function f in $K^m$ therefore represents a binary sequence $f_0, f_1, \ldots, f_{2^m-1}$, where $f_i = \pm 1$. Corresponding to this sequence is the binary sequence of 0's and 1's $f_0, f_1, \ldots, f_{2^m-1}$, obtained by setting

$$\hat{f}_i = \frac{1 + f_i}{2} \quad .$$

In the vector space $H^m$, we can define an inner product

$$(f, g) = \int_0^1 f(x)\, g(x)\, dx \quad , \quad f, g \in H^m \quad ,$$

and a norm

$$||f|| = \left[\int_0^1 f^2(x)\, dx\right]^{1/2} \quad .$$

If f is in $K^m$, then $||f|| = 1$, or any function corresponding to a binary sequence has unit length.

It is easy to show that the inner product (f, g) of two vectors in this representation is equivalent to the Hamming distance in the usual binary representation.

Let $\psi_0, \psi_1, \ldots, \psi_{2^m-1}$ be a set of orthogonal vectors in $K^m$. It follows that any vector in $K^m$ or in $H^m$ can be expanded in a Fourier series,

$$f = \sum_{i=0}^{m-1} (f, \psi_i)\, \psi_i \quad .$$

One set of $\{\psi_i\}$ which forms a basis for $H^m$ consists of the first $2^m$ Walsh* functions. These functions are the completion of the Rademacher functions defined by

$$\phi_o(x) = \begin{cases} +1 & 0 \leqslant x \leqslant 1/2 \\ -1 & 1/2 < x \leqslant 1 \end{cases} \quad , \quad \phi_o(n + x) = \phi_o(x) \quad n = 1, 2, \ldots$$

$$\phi_n(x) = \phi_o(2^n x) \quad .$$

The Walsh functions are then defined by

$$\psi_o = 1 \quad , \quad \psi_n = \phi_{n_1} \phi_{n_2} \cdots \phi_{n_r} \qquad n \geqslant 1$$

$$n = 2^{n_1} + 2^{n_2} + \ldots + 2^{n_r} \qquad n_1 > n_2 > \ldots \qquad n_r \geqslant 0 \quad .$$

For m = 3, the binary sequences corresponding to the Walsh functions are

$$\psi_0 = + + + + + + + +$$
$$\psi_1 = + + + + - - - -$$
$$\psi_2 = + + - - + + - -$$
$$\psi_3 = + + - - - - + +$$
$$\psi_4 = + - + - + - + -$$
$$\psi_5 = + - + - - + - +$$
$$\psi_6 = + - - + + - - +$$
$$\psi_7 = + - - + - + + - \quad ,$$

$$\hat{\psi}_0 = 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1$$
$$\hat{\psi}_1 = 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0 \quad ,$$
etc.,

where the +'s and −'s denote +1 and −1, respectively. These eight orthonormal functions together with their negatives are exactly the first-order Reed-Muller† code‡ $\phi_1^3$. Reed-Muller codes of other orders correspond to certain allowable linear combinations of the $\psi_i$. For example, second-order Reed-Muller codes have the form

$$f = 1/2(\psi_o + \psi_i + \psi_j \psi_i \times \psi_j) \quad .$$

It is hoped that these studies will lead to new codes with fairly simple decoding procedures based upon the computation of Fourier coefficients.

I. L. Lebow

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*J.L. Walsh, Am. J. Math. 55, 5 (1923).

†I.S. Reed, Trans. I.R.E., PGIT-4, 38 (1954).

‡ This correspondence between Walsh functions and Reed-Muller codes has also been observed by J. Dutka and reported in a paper presented to the Boston Section of I.R.E., PGIT, April 1959.

## B. Error-Correcting Coder (ECCO)

The error-correcting encoder and decoder described in the previous quarterly progress report (15 January 1959) has been designed and all required component parts are on order. The encoder occupies two subracks (each 5 × 5 × 17 inches), not including the two power supplies (+6 volts and −8 volts). The decoder occupies three subracks. Each unit will receive and transmit standard Bell data service dipulse signals of sync, timing and data. The encoder receives data at any rate up to 1 kcps and transmits at double frequency.

K. E. Perry

## C. ECCO Shift Registers

The shift register stages for ECCO have been designed to be packaged on a small printed circuit board $1\frac{1}{2} \times \frac{7}{8} \times \frac{1}{4}$ inches. Thirty-two of these small packages or "minicards" will be assembled on a standard 5 × 6 inch plug-in card to form a 32-bit shift register. Figure 52-9 is the circuit diagram of a shift register stage and Fig. 52-10(a) is a photograph of a "minicard." Figure 52-10(b) is a master card containing 32 shift register stages.

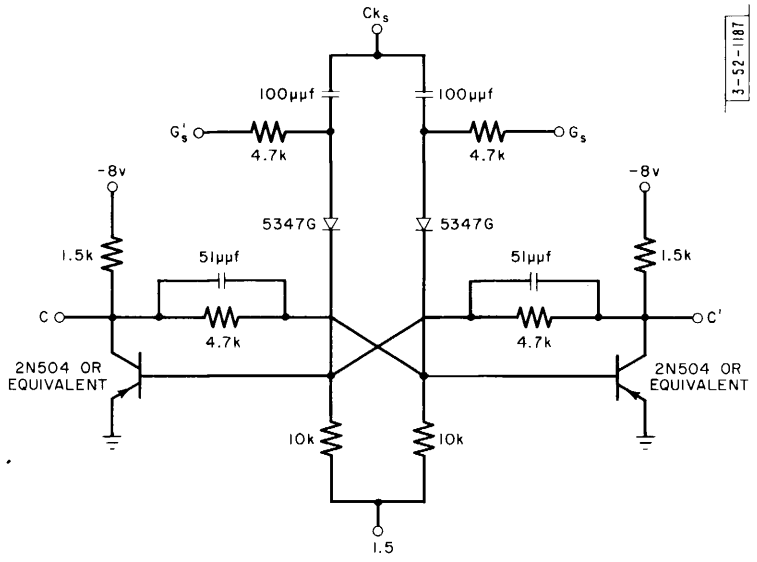


Fig. 52-9. Circuit diagram of a shift register stage.

Each flip-flop is tested commutatively at a prf of 300 kcps with AC and DC loads applied to each side of the flip-flop. The test is carried out with a trigger amplitude of 3 volts, half the value normally used. A number of flip-flops failed to trigger under these test conditions. Since the circuit had been designed to operate with a minimum beta of 9, and all transistors had been selected to have a beta of at least 20, low beta was discounted as a reason for failure. Furthermore, high $I_{CO}$ was not the cause of failure, since transistors with high $I_{CO}$ operated satisfactorily. Experiment showed that failure to trigger was caused by transistors having high values of rise, storage and fall times. To make the circuit trigger reliably when 2N504's are used, triggering and cross-coupling capacitors were increased in value. The original capacitor values are satisfactory when using 2N501's having more carefully controlled response times.

Clock drivers for the shift register will be split up at the rate of one driver for each 32 or 64 flip-flops. The split drivers have been tested and found to have a delay time of 0.05 to 0.1 μsec, depending upon load conditions. The relative pulse timing between drivers differs by only a few millimicroseconds, so no timing problems should be encountered.

D. B. Eshleman

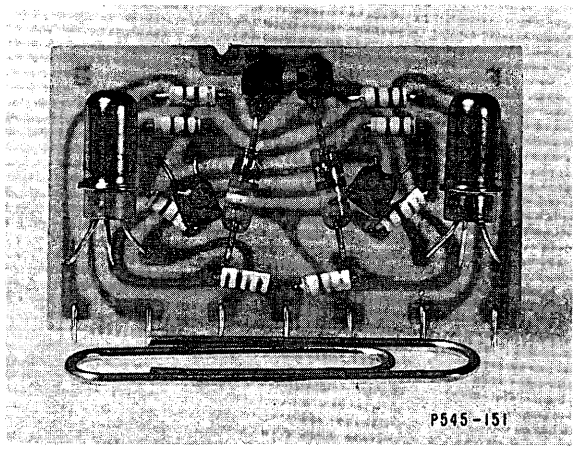


Fig. 52-10(a). ECCO shift register flip-flop minicard.

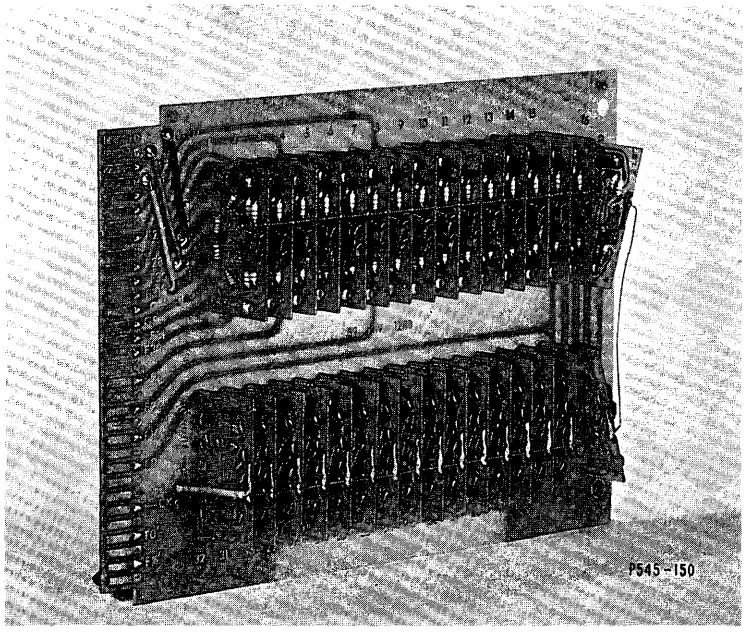


Fig. 52-10(b). Master card of 32 flip-flops.

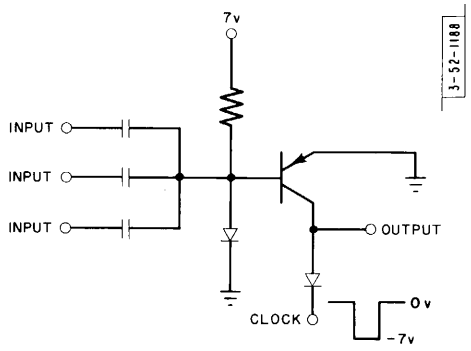


Fig. 52-11. "Or" circuit.

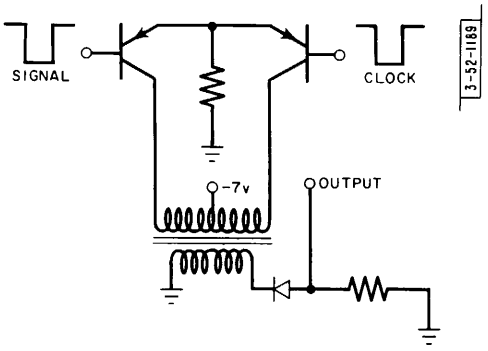


Fig. 52-12. Inverter.

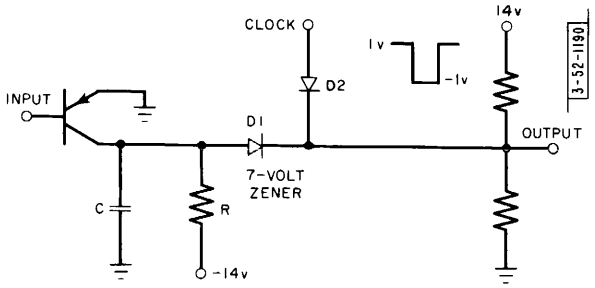


Fig. 52-13. Storage-type inverter.

# IV. COMPUTER COMPONENTS

## A. High-Speed Computer Circuits

Most of the work during the period covered by this report consisted of tests on the capacitor shift register, and designs of associated logic circuits. The capacitor shift register was designed to provide storage in a pulse-no-pulse logic system, and it was described in the Quarterly Progress Report for 15 January 1959.

The advantages and disadvantages of the capacitor shift register turned out to be analogous in many respects to those of a magnetic core shift register. The chief advantages are circuit simplicity and low power consumption. The chief disadvantage is the sensitivity of the circuit to the shape of the clock pulses. If the leading edge of a clock pulse has too short a fall time, a spurious "1" tends to be generated. If the leading edge has too long a fall time a "0" may be propagated when a "1" should be. Moreover, the load presented to the clock pulse generator varies with the information content of the shift register. A shift register full of "0's" presents a very light load to the clock source. A shift register full of "1's" presents a very heavy load to the clock source. This, of course, complicates the problem considerably. Because of the difficulty of pulse shaping at clock rates of 10 megapulses/sec, and higher, this disadvantage is a very serious one if high-speed operation is desired. However, for lower-speed operation, the capacitor shift register might be competitive with the magnetic core shift register in many applications.

A six-stage (three-bit) capacitor shift register was connected in a recirculating loop (output to input), and operated with the clock rate continuously varied from 500 kilo-pulses/sec to 20 megapulses/sec. The circuit operated successfully over this range of clock rates, storing all the combinations of patterns possible with three bits. Above 20 megapulses/sec, certain patterns could not be stored. However, the lower limit was arbitrarily set by the clock pulse generator, which could go no lower.

It has been shown that all logic operations required in computer circuitry can be performed with an inverter (complementing circuit) and either an "and" circuit or an "or" circuit. In pulse-no-pulse logic there are a number of configurations which can serve as "or" circuits. One simple circuit, compatible with the capacitor shift register is shown in Fig. 52-11. However, the design of an inverter is much more difficult. The arrival of a signal pulse at the input of an inverter must cause no pulse to appear at the output. Conversely, the arrival of no signal pulse at the input must cause a signal pulse to appear at the output.

In an effort to obtain a satisfactory inverter circuit, a number of designs have been tried. These fall into two classes. The first class is typified by the differential amplifier configuration of Fig. 52-12. Clock pulses are applied to one base, and signal pulses to the other. For proper operation, the signal pulses must completely envelop the clock pulses both in amplitude and duration. Under these conditions, the simultaneous arrival of a signal pulse and a clock pulse will cause no pulse at the output. However, the arrival of the clock pulse alone will cause an output pulse. In inverters of this class, pulses must be closely controlled with regard to shape, amplitude and arrival time. For this reason, the use of this class of inverters seems undesirable for general high-speed applications.

The second class of inverters involves the use of a storage element such as a capacitor. A circuit illustrating the principle is shown in Fig. 52-13. A signal pulse at the input causes the transistor collector to rise to ground potential, disabling the "and" gate formed by diodes D1 and D2. The next clock pulse, which is delayed with respect to the signal input, finds the

"and" gate disabled, and therefore can cause no output pulse. The time constant RC is so chosen that the "and" gate will be enabled in time to pass the second clock pulse after a signal pulse, providing that another signal pulse does not intervene. A characteristic of this class of inverters is that the output is delayed with respect to the input. At first thought this may appear to be a limitation. However, the rate at which information can be handled is still equivalent to that of the first class of inverters.

The resistor R in Fig. 52-13 can be removed, and the capacitor C can be discharged with the aid of an additional set of clock pulses. Although this approach would permit closer control of timing, the circuit complexity is increased. Also, the extra set of clock pulses makes operation at high speeds more difficult to achieve.

The tests that have been made so far, indicate that storage-type inverters have better margins than the inverters without storage.

R. W. Hofheimer

## B. Digital Display

Development and construction of the transistorized digital cathode-ray-tube display, described in the Division 5 Quarterly Progress Report of 15 January 1959, has progressed satisfactorily.

During the last quarter considerable effort has been expended in developing an efficient technique for transforming our analysis results for the Fourier coefficients to hardware. A harmonic analyzer and a character simulator have been built. These devices will be fully described in a forthcoming technical report. Briefly, the harmonic analyzer is a machine to extract the Fourier coefficients in a 24-point analysis up to the eighth harmonic and the character simulator is a device to set up resistors proportional to the analyzed coefficients in order that the accuracy of the analysis might be checked.

With the above-described approach, we are presently displaying all the lower case letters of the alphabet, ten numbers and miscellaneous symbols. In a $6 \times 6$ inch area, an array of 128 characters by 64 characters is still quite legible.

K. E. Perry
E. J. Aho

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Unclassified

# COMPUTER COMPONENTS
# GROUP 53

GOODENOUGH, J. B., *Leader*
SMITH, D. O., *Assistant Leader*

ARNOTT, R. J.
BARCK, P. E.
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DUGGER, C. O.
DWIGHT, K., Jr.
ECKL, D. J.
FERRETTI, A.
HUBER, E. E., Jr.
JOHNSTON, R. C.
KAPLAN, T. A.
KIRK, C. T.
LARSON, E. G.
MENYUK, N.
SOOHOO, R. F.
WEISS, G. P.
WICKHAM, D. G.
WOLD, A.

Unclassified

# COMPUTER COMPONENTS
# GROUP 53

## I. SUMMARY

### A. Magnetic Materials

The fruitfulness of an explicit consideration of the relative symmetries of the outer-electron wave functions and the near-neighbor configuration for cations or atoms in a crystalline structure appears to have once again been demonstrated. A definite correlation between the strengths of the magnetic A-B and B-B interactions in spinels and the number of cation outer-d electrons has been found. This correlation provides strong evidence for the presence of direct B-B interactions. In order to establish the suggested origin for these correlations and to investigate promising electrical characteristics, a series of compounds containing $3d^{3-n}$ cations is being prepared. These preparations are quite difficult, and considerable effort has gone into the investigation of suitable preparation procedures. The investigation of the crystallographic and magnetic properties of binary nitrides initiated last quarter continues. Although a considerable body of information exists on the preparation and crystallographic properties of the individual transition-metal nitrides, much less information is available concerning binary nitrides. A principal source of difficulty for fundamental studies of the physical properties of ferrites is the preparation of pure, homogeneous samples. A significant improvement in preparation procedures has been developed in this laboratory, and the degree of purity and homogeneity of the samples is being checked by electrical measurements. Spinels containing other magnetic configurations than those proposed by Néel are still poorly understood. Studies on a natural crystal of hausmannite ($Mn_3O_4$) have demonstrated several surprising facts. Further, a theoretical study has shown that in a cubic crystal the Yafet-Kittel angular configuration cannot be ordered even at low temperatures. The proposal of an alternate spin configuration shows promise of providing an interpretation of the interesting, recent neutron-diffraction experiments being performed at Brookhaven National Laboratory, on antiferromagnetic, metallic chromium.

### B. Magnetic Films

Attention continues to be focused on the two parameters that must be controlled in the preparation of large, uniform arrays for computer-memory application: the coercive force for flux reversal by domain-wall motion $H_w$ and the so-called rotational coercive force $H_k = 2k/M$. The experiments this quarter have demonstrated the subtlety of some of the contributions to these parameters. It is hoped that electron-diffraction studies (facilities were made available this quarter) will reveal relationships between these parameters and the structure of the films. In order to investigate the potential of magnetic films as parametron elements, a study of the initial permeability of the films at microwave frequencies has been initiated. The microwave equipment was assembled during this quarter, and measurements have begun.

### C. Semiconducting Components

During this quarter the principal emphasis on semiconducting computer components has been aimed at the testing of and design improvements for the switching properties of the SBT's, MAT's and MADT's currently being used in the TX-2 computer. The theoretical analyses have helped to improve the switching speeds, and attention is now shifting to the requirements placed on devices by UHF computers. The design of the transistor circuits for the transistor-driven core memory (TDCM) is probably the last circuit design Group 53 will perform for the TX-2 computer.

The microwave transistor and the parametron presently appear to be the most promising active elements for UHF computers. A 50-Mcps pulse generator has been built which uses a Sperry S-180 avalanche diode. The cryosar has been investigated as a possible high-density, high-speed memory element.

J. B. Goodenough

## II. MAGNETIC MATERIALS

### A. Inorganic Chemistry and Crystallography

#### 1. Spinels Containing $3d^{3-n}$ Cations

A brief description and interpretation of the magnetic properties of spinels containing transition-metal cations with outer electron configurations $3d^{3-n}$ was given in the preceding quarterly progress report. A systematic study of these materials is in progress. Although several new materials have been prepared and work proceeds on several troublesome syntheses, the ideas which form the basis of this study have been sufficiently developed to become the subject of a short article that has been accepted for publication in The Physical Review.

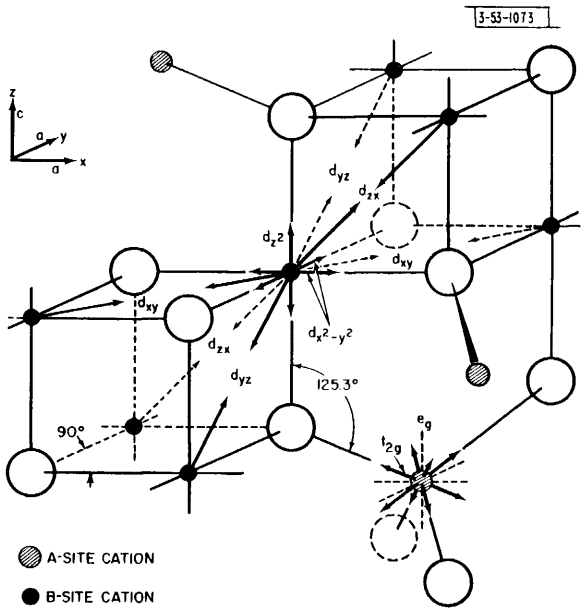


Fig. 53-1. Schematic diagram of cation-orbital orientation relative to the spinel structure.

In metal oxides, the cations are separated by relatively large distances, and the magnetic interactions of principal previous interest have been indirect interactions in which an anion is an intermediary. Such indirect-exchange mechanisms are an optimum if the two cations are located on opposite sides of an anion.[1] An indirect-exchange mechanism and consequent coupling rules have been proposed[2-4] for this case which are at once consistent with recent detailed calculations[5] and with experimental data on perovskite-type and rock-salt-type structures. In the spinel lattice the situation is complicated by the fact that the cations are not located on opposite sides of the anion. The tetrahedral (A) cation-anion-octahedral (B) cation angle is ~125°, the B-anion-B angle is 90° (Fig. 53-1), and the rules developed for 180° angles are not applicable.

Examination of the d orbital symmetry relative to the symmetry of the spinel structure leads to the following postulates:

(a) If both the A-site $t_{2g}$ orbitals and the B-site $e_g$ orbitals are half-filled (or less than half-filled and degenerate), the A-B interactions are strongly antiferromagnetic.

(b) If the A-site $t_{2g}$ orbitals and/or the B-site $e_g$ orbitals are empty, the A-B interactions are relatively weak and antiferromagnetic.*

(c) If the B-site $t_{2g}$ orbitals are half-filled (or less than half-filled and degenerate), direct B-B interactions are possible. The strength of

* For 180° cation-anion-cation angles σ-bonding predominates, but with ~125° angles the π-bonding is expected to dominate the magnetic-exchange interactions.[3]

these antiferromagnetic interactions depends strongly on intercation distance and anion shielding, both of which are reduced if partial covalence via empty $e_g$ orbitals is possible. Thus the condition for relatively strong B-B interactions is compatible with the conditions for weak A-B interactions.

(d) If the $t_{2g}$ orbitals of one or both of two cations in neighboring B-sites are full (or more than half-full and degenerate), no direct B-B interactions are possible.

Whereas Yafet and Kittel[6] have suggested that the atomic moments may not be collinear if the A-B and B-B interactions are comparable, the data of Table 53-I are interpreted on the assumption that these conditions introduce a ferrimagnetic or antiferromagnetic arrangement in the B-sites.*

Further details are available in a forthcoming publication in The Physical Review.

D. G. Wickham
J. B. Goodenough

2. Transition-Metal Nitrides

$Fe_4N$ has been prepared by heating finely divided iron powder at 450°C in a stream of mixed $NH_3$ and $H_2$ in a ratio of 2:5 for five hours. The measured moment in 8500 gauss for the analyzed phase $Fe_4N_{0.96}$ was 1.87 Bohr magnetons per metal atom. This is considerably lower than the value of 2.22 reported by Guillaud and Creveaux.[8] The discrepancy is probably due to the low fields available in the present investigation. There appears to be no equivalent $Ni_4N$ compound. However, compositions in the system $Fe_{4-x}Ni_xN$ have been prepared where x has values between zero and 3.33. Members of this series appear to be isostructural with $Fe_4N$. In the manganese-nitrogen system, a two-phase region consisting of $Mn_3N$ and $Mn_4N$ exists from $x \sim 3.33$ to almost 4. At ratios of Fe:Ni close to 1:1 (i.e., from $FeNi_{1.06}$ to $Fe_{1.04}Ni$), increased nitrogen content produces a new phase having the approximate composition FeNiN. This phase is tetragonal and can be derived from the f.c.c. $Fe_4N$ cell by an expansion of the lattice in the a direction and a contraction in the c direction. This pseudocubic, tetragonal cell has a = 4.003 Å and c = 3.715 Å. A simpler primitive tetragonal cell having a = 2.830 Å and c = 3.713 Å can be selected as shown in Fig. 53-2. Examination of the strong lines of the x-ray pattern indicated that the metal atoms are located on a body-centered tetragonal lattice. It was proposed that the iron and nickel atoms are ordered with iron atoms at the corners of the unit cell and nickel atoms at the body centers. The nitrogen atoms were placed on the centers of the c faces. Calculated intensities for all possible reflections based on this structure are reported in Table 53-II. Intensity calculations based on a model without iron-nickel ordering are also included for comparison. Since no correction was made for temperature effects, the calculated intensities

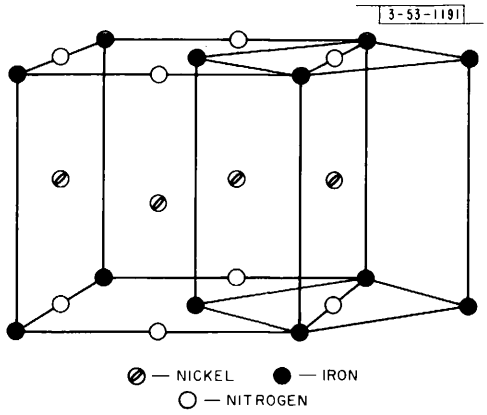


Fig. 53-2. Comparison of the tetragonal FeNiN cell with its pseudocubic $Fe_2Ni_2N_2$ equivalent.

* McGuire[7] has made a similar assumption in the interpretation of his magnetic data for the chromites.

TABLE 53-I

MAGNETIC AND CRYSTALLOGRAPHIC DATA FOR SEVERAL SPINELS

| Compound* | $n_B$ (calculated)† | $n_B$ (observed)† | $T_c$ (°K) | Structure (25°C) | $a_o$ | $c_o$ | Reference |
|---|---|---|---|---|---|---|---|
| $\vec{Co}^{2+}[Co^{III}_{0.9}\overleftarrow{Mn}^{3+}_{1.1}]O_4$ | 1.4 | 1.1 | 190 | Cubic | 8.295 | | 9 |
| $\vec{Co}^{2+}[\overleftarrow{Mn}^{3+}_{1.4}\vec{Mn}^{3+}_{0.6}]O_4$ | 0.2 | 0.1 | 85 | Tetragonal | 8.157 | 9.454 | 9 |
| $\vec{Cu}^{2+}_{0.04}\vec{Fe}^{3+}_{0.96}[\overleftarrow{Cu}^{2+}_{0.96}\overleftarrow{Fe}^{3+}_{1.04}]O_4$ | 1.3 | 1.3 | 728 | Tetragonal | 8.24 | 8.68 | 10 |
| $\vec{Mn}^{2+}[\overleftarrow{Cr}^{3+}_{1.5}\vec{Cr}^{3+}_{0.5}]O_4$ | 1.6 | 1.4 | 55 | Cubic | 8.43 | | 10 |
| $\vec{Mn}^{2+}[\overleftarrow{Fe}^{3+}_{1.5}\vec{Cr}^{3+}_{0.5}]O_4$ | 1.4 | 1.73 | 483 | Cubic | 8.498 | | 10 |
| $\vec{Mn}^{2+}[\overleftarrow{Fe}^{3+}\overleftarrow{Cr}^{3+}_{0.5}\vec{Cr}^{3+}_{0.5}]O_4$ | 0.4 | 0.25 | 370 | Cubic | 8.481 | | 10 |
| $\vec{Fe}^{2+}[\overleftarrow{Cr}^{3+}_{1.5}\vec{Cr}^{3+}_{0.5}]O_4$ | 1.0 | 0.8 | 90 | Cubic | 8.392 | | 11 |
| $\vec{Co}^{2+}[\overleftarrow{Cr}^{3+}_{1.5}\vec{Cr}^{3+}_{0.5}]O_4$ | 0 | 0.1 | 100 | Cubic | 8.32 | | 7 |
| $\vec{Ni}^{2+}_{1-x}\overleftarrow{Ni}^{2+}_{x}[\overleftarrow{Cr}^{3+}_{1+y}\vec{Cr}^{3+}_{1-y}]O_4$ | — | 0.1 – 0.3 | 70 | Tetragonal | 8.24 | 8.44 | 7, 11 |
| $\vec{Fe}^{3+}[\overleftarrow{Fe}^{2+}\overleftarrow{Cr}^{3+}]O_4$ | 2.0 | 2.4 | 473 | Cubic | 8.396 | | 12, 13 |
| $\vec{Fe}^{3+}[\overleftarrow{Co}^{2+}\overleftarrow{Cr}^{3+}]O_4$ | 1.0 | 1.0 | — | Cubic | 8.35 | | 14 |
| $\vec{Fe}^{3+}[\overleftarrow{Ni}^{2+}\overleftarrow{Cr}^{3+}]O_4$ | 0.2 | 0.4 | 690 | Cubic | 8.301 | | 14, 15 |
| $\vec{Mn}^{2+}[\overleftarrow{Mn}^{2+}\vec{V}^{4+}]O_4$ | 0.6 | 0.7 | — | Cubic | 8.579 | | 16 |

* Postulated chemical formula.

† $n_B$ is magnetization in Bohr magnetons per molecule. Calculations assume spin-only atomic moments except for A-site $Mn^{2+}$ ($4.6\mu_B$) and B-site $Ni^{2+}$ ($2.2\mu_B$).

TABLE 53-II

COMPARISON OF OBSERVED X-RAY INTENSITIES FOR FeNiN WITH ORDERED AND NONORDERED MODELS

| No. | h-k-l | d | $I_{observed}$ Integrated Intensities | Ordered Metal Atoms | No Ordering |
|---|---|---|---|---|---|
| 1 | 001 | 3.7041 | 0.5 | 0.8 | 1.95 |
| 2 | 100 | 2.8240 | 5.3 | 3.1 | 1.53 |
| 3 | 101 | 2.2485 | 100. | 100. | 100. |
| 4 | 110 | 1.9994 | 52. | 51.1 | 50.7 |
| 5 | 002 | 1.8552 | 19. | 19.6 | 19.4 |
| 6 | 111 | 1.7616* | — | 0.15 | 0.5 |
| 7 | 102 | 1.5512 | 0.9 | 0.7 | 0.3 |
| 8 | 200 | 1.4148 | 12.5 | 16.3 | 16.7 |
| 9 | 112 | 1.3610 | 24.8 | 30.1 | 30.8 |
| 10 | 201 | 1.3301* | — | 0.04 | 0.2 |
| 11 | 2.0 | 1.2658 | VVW | 0.4 | 0.2 |
| 12 | 003 | 1.2372 | — | 0.005 | 0.04 |
| 13 | 211 | 1.1980 | 24.9 | 36.3 | 37.0 |
| 14 | 103 | 1.1344 | 14.4 | 19.0 | 19.2 |
| 15 | 202 | 1.1254 | 18.5 | 26.4 | 26.7 |
| 16 | 113 | 1.0524* | — | 0.03 | 0.2 |
| 17 | 220 | 1.0007 | 14.8 | 26.3 | 26.6 |

*Calculated intensities; lines not observed.

Note: All intensities, both observed and calculated, were normalized to make the strongest line (No.3) = 100.

tend to deviate from the observed values for small d spacings. However, the obvious agreement of the ordered model with the observed intensities indicates strongly that this is the preferred structure.

A. Wold
R.J. Arnott
R.W. Small

## B. Ceramics

### 1. The System $NiMn_2O_4$-$Mn_3O_4$

The composition, crystallography and electrical properties of materials in this spinel system were described at some length in the preceding quarterly progress report. An interpretation of the magnetic properties has been deferred until further measurements could be made on several of the compositions. A close-to-stoichiometric $NiMn_2O_4$ has finally been prepared. Nickel oxide and $Mn_2O_3$ were obtained in a finely divided condition by calcination of $NiCO_3$ and $MnCO_3$. Accurately weighed equivalent quantities were very thoroughly ground together and the mixture fired for several days at 900°C with interruptions for regrinding. Finally, the product was heated at 1000°C for three days in an air atmosphere, slowly cooled to 600°C, and quenched to room temperature. X-ray analysis showed a single, cubic-spinel phase. Chemical

analysis for the oxidation value gave 47.06 ± 0.11 per cent $Mn^{3+}$ compared with the theoretical value of 47.24 per cent. The magnetic moment at 4°K in a field of 10,000 oe is $0.8\mu_B$/molecule.

E. G. Larson

2. Conductivity of Stoichiometric Ferrites

The conductivity of stoichiometric ferrites is being studied. These measurements, although indicative of the purity of the specimens, are yet (for certain materials) greatly affected by thermal history. The effect of chemical inhomogeneities is clearly demonstrated. The details of these studies will be available shortly.

D. G. Wickham
E. G. Larson

## C. Magnetic Properties of Hausmannite ($Mn_3O_4$)

1. Quasi-Static Properties

A study of the properties of single-crystal hausmannite ($Mn_3O_4$) below its Curie point was begun during the previous quarterly period. Most of the measurements then were taken on the (100)-faced disk at 4.2°K. Preliminary measurements on an (001)-faced disk were reported, but further investigation indicated this sample to have been misoriented. A new (001) sample slice was cut after a careful x-ray determination of the plane, and the anisotropy, magnetization curves, coercive force and dynamic properties of this crystal are now being investigated.

a. Anisotropy

The anisotropy within the (001) plane has twofold symmetry. This agrees with Kasper's neutron diffraction study of $Mn_3O_4$ at 4.2°K, in which he found orthorhombic magnetic symmetry.[17] The easy magnetization direction lies in this plane in the [100] direction. (This is the tetragonal [100] direction, which corresponds to the [110] direction of a cubic spinel structure.) This result, in conjunction with the results reported in the last quarterly progress report on the (100)-faced crystal indicates that the c-axis ([001] direction) of hausmannite is the hard-magnetization direction of the crystal.

The anisotropy constant within the (001) plane at 4.2°K was found to be

$$K_{(001)} = 1.2 \pm 0.3 \times 10^6 \text{ ergs/cm}^3 \quad .$$

The temperature dependence of $K_{(001)}$ has also been investigated. Up to 25°K, the variation of $K_{(001)}$ is within the uncertainty of measurement. Above this temperature a slow reduction of $K_{(001)}$ with increasing temperature is observable, but at 38°K ($T = 0.9T_c$) $K_{(001)}$ is still half its liquid helium value.

b. Magnetization Curves

The variation of magnetization with temperature has been observed at external-field values varying from zero (remanent magnetization) to 12,000 oe. Low-field measurements yield a ferromagnetic Curie temperature $T_c = 42.2 \pm 0.2$°K.

c. Coercive Forces

At 4.2°K, the coercive force of $Mn_3O_4$ is approximately 2800 oe, as shown in the previous quarterly. This is the field $H_c$ required to reduce the net magnetization to zero in the easy magnetization direction. This result is still true on the (001)-faced sample. However, if we consider

the applied field necessary to move domain walls, the situation is more complicated. If we start with the crystal in the demagnetized state and apply an increasing field in the easy magnetization direction, we find that considerable 180° wall motion starts at about 2300 oe. On the other hand, after saturating the crystal by applying a field of ~10,000 oe, a field of approximately 2800 oe is required to induce a sizable 180° wall motion effect. This difference is presumably due to domain nucleation. Finally, 90° wall motion has been studied by first saturating the crystal in the easy direction, and then applying the external field at 90° from the remanent magnetization. A field of 4600 oe is required to induce 90° wall motion. After saturation, this direction becomes the easy magnetization direction of the crystal. These various wall motion "coercive fields" are indicated in Fig. 53-3. Since the crystal is magnetically orthorhombic, it follows that the 90° wall represents a magnetic twin plane rather than a conventional Bloch wall, with large energy differences possible.

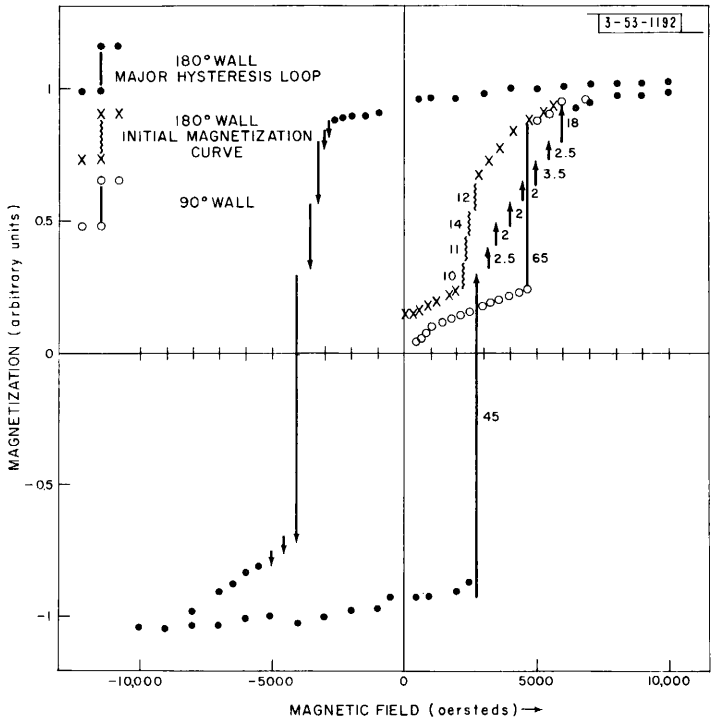


Fig. 53-3. Hysteresis loop and coercive field for 90° and 180° wall motion under various experimental conditions (numbers along vertical lines indicate time required in minutes for magnetization change shown).

The temperature variation of the wall coercive field has also been investigated. Above 4.2°K the coercive field of both 180° and 90° walls decreases precipitously with increasing temperature. By 20°K, the field is down by a factor of twenty to thirty. This change is startling in view of the relative temperature insensitivity of magnetic anisotropy.

2. Dynamic Properties

The study of the time variation of magnetization as a function of applied field has been limited thus far to 4.2°K and 180° wall motion. During a magnetization reversal, three distinct regions appear, each characterized by a different functional dependence of magnetization upon time. In the region responsible for most of the magnetization change $\Delta M$ (~70 to 80 per cent), $\Delta M = A + B \log t$.

The coefficient B varies only slightly with applied fields between 2650 and 4000 oe, despite the fact that the reversal time changes by more than two orders of magnitude over this range.

N. Menyuk
K. Dwight, Jr.

## D. Theory of Yafet-Kittel Angles

### 1. Question of Stability of Yafet-Kittel Angles

Yafet and Kittel[6] showed that, for ferrites with large enough antiferromagnetic A-A and/or B-B interactions (compared with the antiferromagnetic A-B interaction), certain triangular spin arrangements would be preferred (according to molecular-field theory) over the conventional collinear configuration. So far as we know there has been no suggestion that there might exist other configurations of still lower exchange energy. Our recent calculations have shown that, in general, the Yafet-Kittel configurations are not minimum energy states.

The method of calculation is simply to consider the change $\Delta E$ in the classical Heisenberg energy when the spins deviate by small (but arbitrary) vectors from their values in the configuration (C) of interest; if $\Delta E < 0$, then C is said to be unstable (i.e., it is not the correct ground configuration).

### 2. Yafet-Kittel Angles and Spin Waves

On Pickart's[18] suggestion, we began an investigation of the spin wave properties of systems expected to exhibit Yafet-Kittel angles. The hope was that the results might suggest a new experiment that would help decide the "angles" question.

Although these calculations are in a preliminary stage, the results are sufficient to show that the behavior of spin waves in the presence of triangular-spin arrangements (when the latter are stable) will be markedly different from that for a collinear ground state. For example, for a particular Yafet-Kittel configuration on the spinel lattice (which we find stable if there is sufficient tetragonal distortion), the spectrum (magnon frequency $\omega$ vs wave vector k) will have two acoustic branches, one proportional to $k^2$, the other linear on k (for k along the c-axis and small). (We are assuming ferrimagnetism, i.e., the total spin is nonzero.)

### 3. Instability in Collinear Spin Systems

Because of the complexity of the Yafet-Kittel angles, we have found it difficult to obtain intuitive insight into the stability problem, particularly with respect to the question of the ground configuration if the Yafet-Kittel arrangement is unstable. Hence we considered some simple antiferromagnets (linear chain, body-centered-cubic, etc.) which could be made unstable by proper choice of the interaction parameters.

3-53-1193

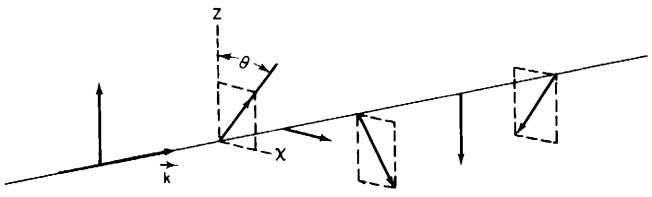


Fig. 53-4. Configuration $C_1$ (for linear chain), only A spins are shown.

Let $C_o$ be the lowest configuration when only nearest-neighbor forces are considered. We were able to guess a class C of equilibrium configurations (Fig. 53-4), which will always include a state of lower energy than $C_o$ when the latter is unstable.

An interesting property of C is that in the b.c.c., for ratios of second and third neighbor to nearest-neighbor interactions of ~0.6 and 0.1, respectively, the lowest configuration will exhibit a striking neutron diffraction pattern similar to that recently observed with a single crystal of chromium.[19] Further study is being made.

T.A. Kaplan

## III. MAGNETIC FILMS

### A. The Physical Origins of $H_k$ and $H_w$

#### 1. Inverted Films

In the last quarterly progress report the conjecture was made that the anomalous properties of inverted ($H_w/H_k > 1$) films are due to low density. The exact experimental procedures necessary to measure density by direct weight and thickness determination have been worked out. Relative densities of 1000 Å films to ±2 per cent (absolute density to ±5 per cent) can now be measured. An appropriate series of inverted films is being measured.

#### 2. Anisotropy

##### a. Angle-of-Incidence Effect

For films made on a substrate with the normal inclined to the vapor beam, an easy axis develops perpendicular to the incident beam.[20] Plane polarized light transmitted through these films becomes elliptical and the major axis is rotated. The ellipticity and rotation depend on the relative orientation of the plane of polarization and the film, i.e., the effect is anisotropic in the same way as the magnetic anisotropy. It is likely that this effect is the same as that reported by von Kundt[21] in 1886, although the association with the angle-of-incidence was not made by von Kundt.

In order to study this effect quantitatively, a high-quality ellipsometer will be required. Such instruments are not much in demand and consequently difficult to purchase commercially.

##### b. Isotropic Films

As a means of identifying the origins of film anisotropy attempts are continuing to make isotropic films. Deposition in a rotating-circular field eliminates the short-range directional order of iron pairs, and the angle-of-incidence has been reduced to ~1° by using very narrow apertures and a moving substrate to index each spot successively over the aperture. A subtle effect has appeared, depending on whether the substrate is moved continuously or discontinuously. For continuous motion, an easy axis develops perpendicular to the direction of motion ($H_k$ ~ 1 oe for 83-17). It seems unlikely that real kinetic effects are present, although attribution of the effect to angle-of-incidence also seems implausible. For spots separately registered over the aperture, the anisotropy is of the same order of magnitude as for the moving case, but the direction is now random from spot to spot.

The general conclusion that can be reached from the above experiments is that very slight asymmetries in experimental conditions, as for example, source emission, substrate tilt or

motion, or alignment of source and substrate, can give rise to structure anisotropy and consequent magnetic anisotropy. This line of reasoning will be explored in the next quarter.

D. O. Smith
E. E. Huber, Jr.
G. P. Weiss

## B. Some Microwave Properties of Thin Films

### 1. Perturbation Theory

Consider the case of a film placed against the back wall of a $TE_{10n}$ microwave cavity as shown in Fig. 53-5. If the easy axis of the film is oriented along the y-direction, then the initial susceptibility of the film $\chi_i = \chi_i' - j\chi_i''$ can be shown, by use of perturbation theory, to be given in emu by the following expressions:

$$4\pi\chi_i' = -\frac{\Delta\omega_r}{\omega_o}\left(\frac{b}{b'}\right)\left(\frac{d}{\delta}\right)\frac{a\left[1+\left(\frac{d}{na}\right)^2\right]}{a' + \frac{a}{2\pi}\left[\sin\frac{2\pi x_o}{a} - \sin\frac{2\pi(x_o+a')}{a}\right]}$$

$$4\pi\chi_i'' = +\Delta\left(\frac{1}{2Q}\right)\left(\frac{b}{b'}\right)\left(\frac{d}{\delta}\right)\frac{a\left[1+\left(\frac{d}{na}\right)^2\right]}{a' + \frac{a}{2\pi}\left[\sin\frac{2\pi x_o}{a} - \sin\frac{2\pi(x_o+a')}{a}\right]}, \qquad (1)$$

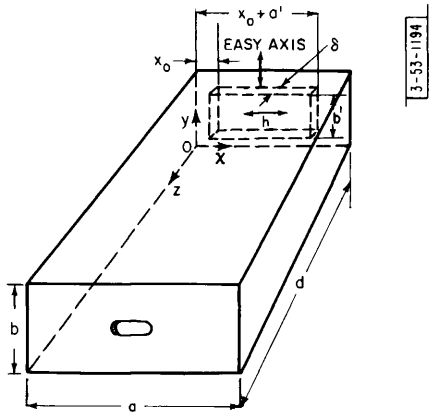


Fig. 53-5. Microwave cavity loaded with a Permalloy film.

where $\Delta\omega_r/\omega_o = (\omega - \omega_o)/\omega_o$ is the ratio of the change in the resonance frequency due to the presence of the film divided by the resonance frequency of the empty cavity and $\Delta Q$ is the corresponding change in Q.

Equation (1) was derived by assuming negligible eddy current effects. Expression (1) is also valid for the case where a DC field is applied along the y-axis.

### 2. Eddy Currents and Exchange Effects

For a medium with a real scalar permeability $\mu$ and resistivity $\rho$ the magnetic field $\vec{h}$ inside the medium is given approximately by

$$h \simeq h_s \frac{1 + 0.0208\,(2z'/S)^4}{1 + 0.0208\,(\delta/S)^4} \quad . \qquad (2)$$

For example, if $\mu_i = 10$, $\rho' = 21 \times 10^{-6}$ ohm-cm, $\delta = 3900$ Å, and $f = 9 \times 10^9$ cps, the ratio $h_s/h$ at the center of the film is only 1.0013. Thus, there is practically no variation of the magnitude of the magnetic field across the thickness of the film, and therefore the neglect of eddy-current effects in the derivation of Eq. (1) is completely justified.

Because of the small variation in h, exchange effects can also be neglected. Although $h_s$ of Eq. (2) has an x-dependence, the variation is so slow on the microscopic scale that the exchange effects between adjacent spins in the x-direction can again be ignored.

At microwave frequencies, the initial permeability is practically real and can be shown to be given simply by

$$\mu_i \simeq \mu_i' \simeq 1 - \frac{(\gamma 4\pi M)^2}{\omega^2} \quad . \qquad (3)$$

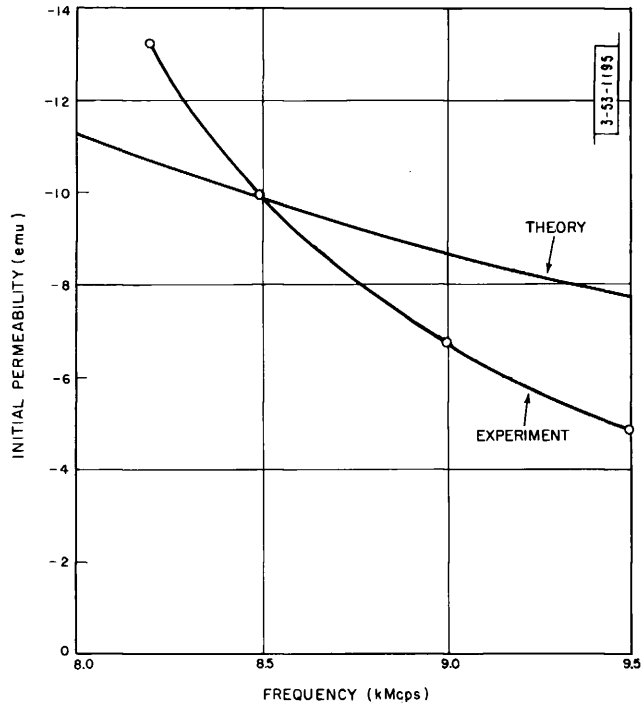


Fig. 53-6. Initial permeability of Permalloy vs frequency.

Equation (3) is plotted in Fig. 53-6 for a range of frequencies at X-band. The experimental values of $\mu_i' = 1 + 4\pi\chi_i'$ are obtained from Eq. (1) by multiplying the measured $\Delta\omega_r/\omega_o$ by the pertinent geometrical factors. These experimental points are also given in the figure for comparison with the theoretical values. The agreement is seen to be reasonably good except that the experimental susceptibility seems to decrease somewhat faster than the $1/\omega^2$ dependence of Eq. (3). This is probably due to the fact that the film is placed against the back wall of the cavity, rather than at the center of the cavity, so that the RF magnetic field at its two surfaces cannot be exactly equal.

R. F. Soohoo

## IV. SEMICONDUCTING COMPONENTS

### A. Cryosar-Diode Circuits

The previous two quarterly progress reports have discussed frequency measurements on a cryosar diode and some logic circuits employing delay lines for pulse storage. During this past quarter pulse measurements were made on a smaller, faster diode, and the random-access memory applications of the cryosar were analyzed.

A diode with an electrode diameter of 0.005 inch and a wafer thickness of 0.005 inch was obtained from Dr. R. H. Rediker of this Laboratory. The test setup shown in Fig. 53-7 was assembled. The input to the diode consists of two pulses of opposite polarity with variable width and spacing. It required a 10-mμsec pulse of 0.37 volt to change its state (to either on or off) or a 2-mμsec pulse of 0.49 volt. The rise time of the pulses of about 1.5 mμsec could have accounted for this difference. From the data taken, it is believed that the device would operate satisfactorily on 2-mμsec pulses spaced by 3 mμsec.

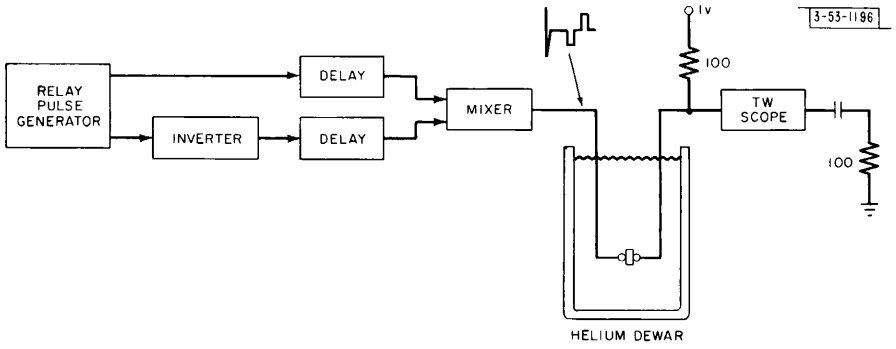


Fig. 53-7. Double-pulse-cryosar test.

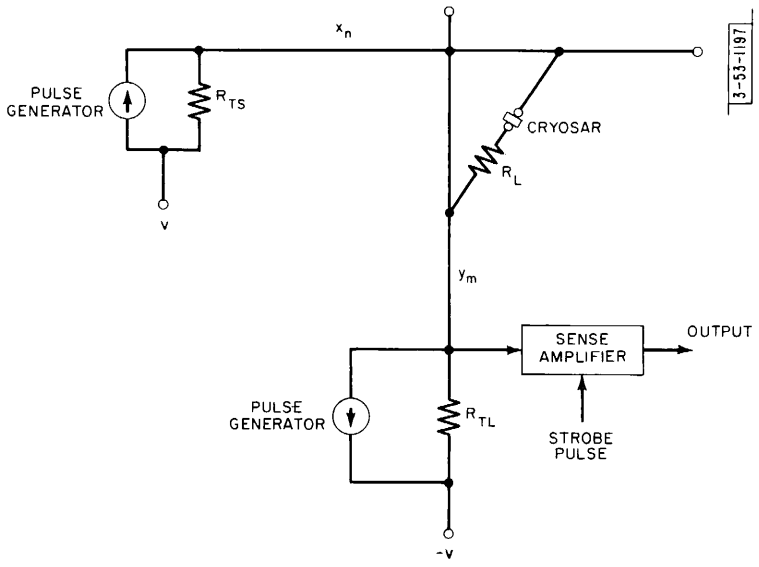


Fig. 53-8. Matrix-memory circuit.

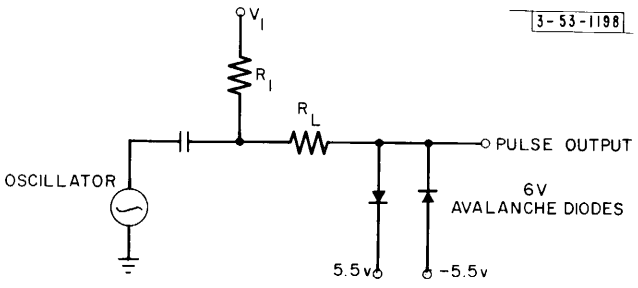


Fig. 53-9. 50-Mcps pulse generator.

An analysis has been made of a coincident-voltage matrix memory using the cryosar. Figure 53-8 shows one bit of an n-by-m matrix containing (nm) bits. To write information, the lines are pulsed in opposite directions. To read, an x-line is pulsed and the voltage on a y-line observed with a sense amplifier. The one-zero ratio at the sense amplifier is reduced by all the "sneak" paths. This effect was analyzed with the result

$$S = \frac{1}{R_{TL}R_{TS}nm/R_L^2 + R_L/R_0} ,$$

where S is the one-zero ratio, $R_0$ the "off" resistance of the cryosar, and the remaining symbols are as shown in Fig. 53-8. If numbers are substituted in the above equation, it appears that a memory of the order of 25,000 to 100,000 is possible based on this consideration. For these large sizes, however, the output voltage is quite small and the $R_L$ of around 10 K might slow down the switching time.

### B. Avalanche Diode Circuits

The Sperry S-180 avalanche diode appeared to have high-speed applications in view of its quoted rise time of 0.15 mμsec. The sine-wave clipping circuit shown in Fig. 53-9 produced sharp rectangular pulses at frequencies up to at least 50 Mcps. A number of measurement problems prevented the finding of the ultimate limit due to the diodes. The diodes are never biased forward, so have no recovery problems. The components $R_1$ and $V_1$ are a bias source which shifts the clipping level and gives a rectangular instead of a square-wave output if so desired. The circuit might have applications as a clock for a 50- to 100-Mcps computer, as a three-phase pulse source for parametron, Esaki diode and cryosar-diode logic circuits, and for general pulse testing.

### C. Circuits for the TX-2 Transistor-Driven Core Memory

The TX-2 "T" and "U" memories are conventional coincident-current, random-access, destructive-read core memories, each with a capacity of 4096, 37-bit words. The 1-μsec 250-ma current pulse for read and write selection is entirely derived from transistor and diode circuits.

A prototype memory[22] was completed in 1957 for the TX-0 computer and is presently in operation. The n-p-n Sylvania 901 symmetrical-core-driver transistor used in the TX-0 memory was discontinued by the manufacturer. The new memories employ the GSN-101 p-n-p core driver, developed by the Philco Research Division to meet our requirements. The use of other new transistors, such as the Philco L-5432, a high-voltage, microalloy, diffused-base transistor, cut the transistor count in the basic memory from 1406 to 1028. The Philco 2N601 p-n-p alloy transistor also greatly simplifies the read-write generator design. A report on this memory with detailed circuit diagrams will be prepared next quarter.

### D. High-Temperature Storage Tests on Microalloy Transistors

The results of an elevated-temperature storage test at 104°C on 16 type 2N393 transistors were published in the Quarterly Progress Reports on Solid State Research for 1 August and 1 November 1958.

It was intended to run further tests at 85°C, the temperature specification limit of the transistor. However, in the meantime a change was made in the design of the collector region of the transistor which is stated by the manufacturer to have eliminated the punch-through degradation problem.

We have obtained some better temperature controls than were used previously and have started new tests with 40 transistors at 100°C and an equal number at 85°C. First results will be included in the next quarterly progress report.

R. L. Burke
D. J. Eckl

## E. Transistor Switching Theory

During this quarter, theoretical calculations of base layer switching times were carried out for several types of impurity distribution functions in the base region. These calculations were made on the basis of a general solution of the one-dimensional time-dependent minority-carrier diffusion equation described in Lincoln Laboratory Group Report No. 53-15 (not generally available).

The results of these calculations show that the value of diffusion techniques in the fabrication of UHF switching transistors is not due to improved frequency response resulting from the so-called "built-in field" from diffusion, but to the very small base width which can be achieved by the diffusion technique with simultaneous, separate control over the impurities: i.e., the total amount of impurities in the base as well as the base-region impurity density at both the emitter and collector junctions.

C. T. Kirk

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Unclassified

# PATTERN RECOGNITION
# GROUP 54

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Unclassified

# PATTERN RECOGNITION
# GROUP 54

## I. INTRODUCTION

The MAUDE Morse Translator has been rewired to include a variable delta for spaces; one for marks is nearly complete. We are turning our attention to the front end – the demodulator – which must produce a suitable binary signal for the digital processes of MAUDE. Past work, for example, on the filter-limiter-filter receiver, suggests that this is a substantial problem.

A pandemonium approach is being programmed on the 709 for character recognition. Several elementary hill-climbers, or multiple parallel optimizers have been built and tested.

O. G. Selfridge

## II. MAUDE

### A. MAUDE Tests

We have been incorporating variable deltas for marks and spaces into MAUDE, a Morse Automatic Decoder.

The distribution of mark durations is illustrated in Fig. 54-1. Aside from those decisions made directly by the various "rules of 6," the alphabet test and the error sign detector, all decisions are based on the threshold T; T is $\Delta$ greater than $\Theta$, the running average of marks selected by the minimum rule of 6. In earlier models of MAUDE, $\Delta$ was fixed at a value that would process Morse signals best (on the average). This technique did not result in adequate performance because the distributions illustrated in Fig. 54-1 vary too widely among operators. We have tested a "variable delta" on a 709 simulation program, both for marks and spaces, with encouraging results.

The action of the discriminator for the variable delta threshold is shown in Fig. 54-2. The digital discriminator output adds to the threshold T.

Any mark not picked by a rule of 6 which falls to the left of T, within the discriminator gate, is presumed to be a long dot, and causes $\Delta$ to increase, thus pushing T to the right. A mark to the right of T correspondingly moves T to the left. However, any mark picked as the minimum of 6, which falls anywhere in the discriminator window on either side of T, increases $\Delta$ and any mark picked as the maximum of 6 similarly decreases $\Delta$. Thus, if T should stray too far from the null point, the actions of the rules of 6 would bring it back.

We also note that knowledge of $\Theta$ localizes the possible values of $\Delta$. Empirically, we have found that the $\Delta$'s for both marks and spaces range over duration ratios of about 3 to 1.

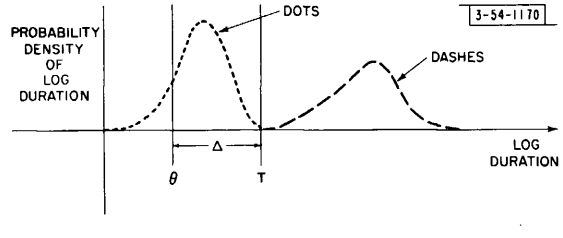


Fig. 54-1. Distribution function for mark durations.

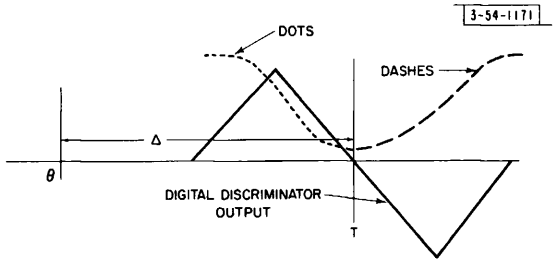


Fig. 54-2. Threshold control for variable delta.

The above system has been tested on the symbol-space-letter-space threshold and, as predicted by previous runs on the simulation program, works very well. Similar circuitry for the marks is now being built.

R. S. Berg
B. Gold
W. C. McLaughlin

## B. MAUDE Demodulator

We are studying the demodulator needed for MAUDE: For our purposes this includes roughly everything needed to produce a clean binary Morse signal from the RF energy. It must allow for many factors:

(1) Fading is prevalent in the MF and HF bands and can cause fluctuations in the signal levels of 20 or 30 db.

(2) Signal-to-noise ratios may be low in any case.

(3) Adjacent and overriding signals, both natural and man made, cause further deterioration in the signal-to-noise ratio.

(4) Man-made signals such as teletype to some extent resemble Morse signals and compete with them for attention.

(5) Transmitters drift in frequency. The bandwidth of a clean Morse signal is usually less than 100 cps. If the demodulator in fact uses 100-cps-wide filters, ordinary drift rates of cycles per second or tens of cycles per second will imply frequency readjustments several times a minute.

(6) Transmitters more often than not do not produce "clean" binary signals. Anomalies like "key-clicks" or "chirps" are common (Fig. 54-3).

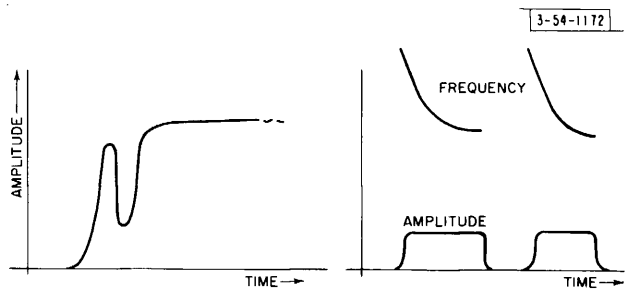


Fig. 54-3. Key click and chirping.

We may further ask the equipment not only to track the signal but also to acquire it: We might specify, perhaps, some 10-kcps frequency band and some call sign.

At present possible attacks are being programmed on the computer and studied on paper.

O. G. Selfridge

## C. Demodulator Statistics

Suppose $x_0(t)$ is white noise and $s_0(t)$ is the real part of $a(t) \exp [iwt + i\varphi] m(t)$ where m is a Morse code signal (i.e., takes only the values 0 and 1). $y(t) = x(t) + s(t)$ is the result of passing $x_0 + s_0$ through a filter whose spectrum is 1 for some frequency interval and 0 elsewhere. We wish to test the hypothesis that $a = 0$ against the compound hypothesis that $a \neq 0$. For given a, $\varphi$, w and m, the log likelihood ratio over the interval 0, T is

$$\int_0^T y(t)\, s_0(t)\, dt - \int_0^T s_0^2(t)\, dt \quad .$$

Maximizing this on $\varphi$ gives

$$\int_0^T y(t)\, s_0(t)\, dt - \int_0^T s_0^2(t)\, dt \quad .$$

If a is assumed constant, the expression can be further maximized to give

$$\frac{1}{2} \frac{\int_0^T y(t)\, e^{iwt}\, m(t)\, dt^2}{\int_0^T m^2(t)\, dt} \quad .$$

If the noise has a variable multiplier which is assumed constant on 0, T, the above ratio is to be divided by $\Theta$. Inserting estimates for $\Theta$ and m in this ratio, maximizing over w, and comparing the result with a fixed threshold give a Neyman-Pearson test for this problem.

T. S. Pitcher

## III. LEARNING

### A. Pandemonium Studies

Input data are assumed to take the form of an $N \times N$ array of bits representing a quantized two-dimensional pattern. The problem is to decide, for any such input pattern, which of a set of P possible "learned" patterns it is (including a "can't decide," if desired).

The approach has been as follows. Some appropriate tests are invented, the application of any one of which to any exemplar of the P possible patterns will produce only a small number of outcomes (pats). The tests are many-to-few transformations of the $2^{N^2}$ conceivable input configurations onto a small set of pats. All of the tests are applied to all of a set of labeled examples representing adequately the class of P patterns to be recognized. The results of the tests are tallied as shown in Table 54-I.

TABLE 54-I

| | | |
|---|---|---|
| Test 1 | | |
| Pat 1 | | |
| | Frequency of Pat 1 under Test 1 for Pattern 1 | F(1,1,1) _____ |
| | . | . |
| | . | . |
| | . | . |
| | Frequency of Pat 1 under Test 1 for Pattern P | F(1,1,P) _____ |
| Pat 2 | | F(2,1,1) _____ |
| . | | . |
| . | | . |
| . | | . |
| Test 2 | | F(1,2,1) _____ |
| . | | . |
| . | | . |
| . | | . |

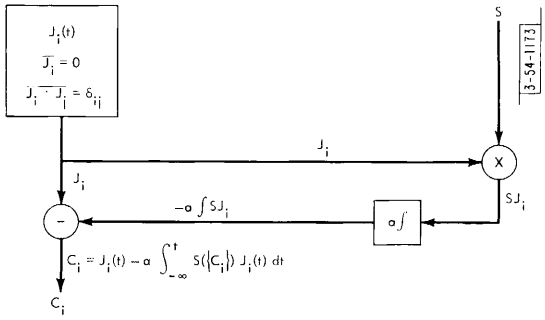


Fig. 54-4. Elementary hill-climber.

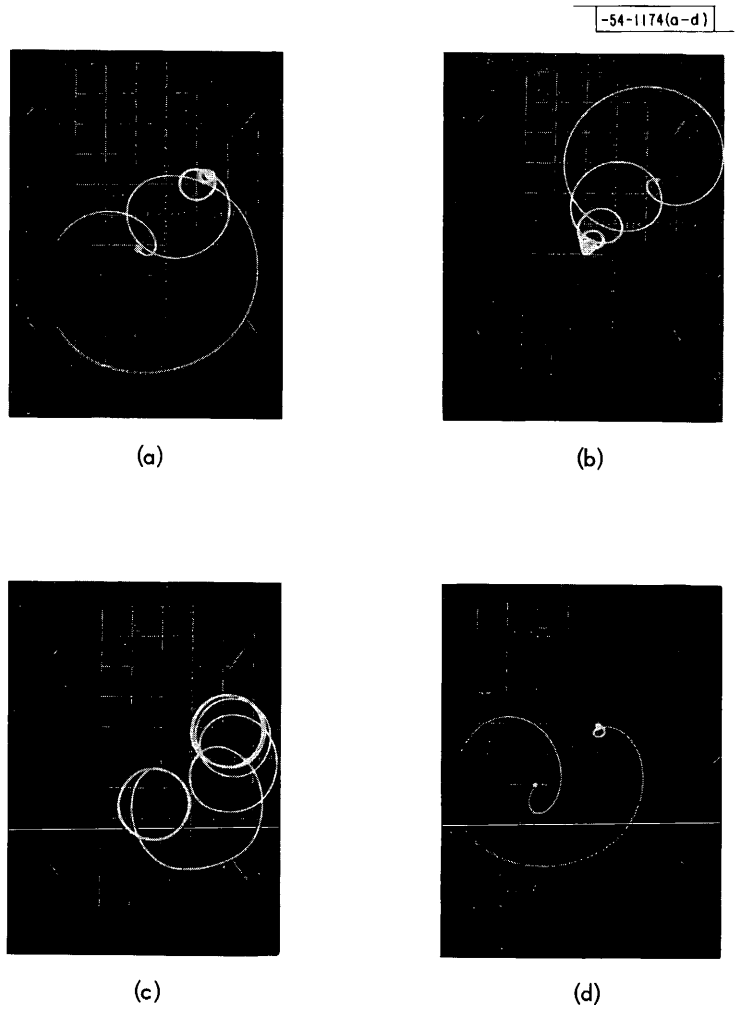


Fig. 54-5. Some two-dimensional trajectories.

An entry F(i, j, k) of the table may now be considered to represent the probability, Pat i having occurred under Test j, that the input pattern was Pattern k. An unknown configuration is then to be identified as follows: Evaluate the P summations

$$\sum_j F(i(j), j, k) \quad , \tag{1}$$

where i(j) is the result of applying Test j to the configuration. Identify the unknown as that k for which the summation is the greatest. If the race is close, an equivocal identification or a "can't decide" may be taken as the result.

Some modifications will be considered later. For example, if a test is good or poor as a whole, its contributions to the decision sums may be increased or decreased by an adjustable factor: We write, instead of (1),

$$\sum_j \alpha_j F(i(j), j, k) \quad . \tag{2}$$

This scheme is being applied to the recognition of hand-printed English capitals as an example. So far it has been programmed for the 709 up through the gathering of data from labeled examples and the construction and debugging of several tests.

W. Doyle

### B. Hill-Climbing

We have been testing certain simple mechanisms to optimize controls in a very general way. We assume we have some system whose state is characterized by the position $C_i$ of certain controls which uniquely determine some positive function S, corresponding to how well the system is doing. We may wish to maximize S, but we lose no generality in minimizing it, so that it corresponds roughly to an error.

In general, it is desirable to adjust all controls simultaneously, that is, to examine the effects of all of them individually and continually to adjust them accordingly. Figure 54-4 shows an easy way to compute $\delta S/\delta C_i$ and modify $C_i$ so as to reduce S. The controls are jittered by a set of orthogonal functions of mean 0 which are correlated with the behavior S.

A variety of such devices has been built and tested; their behavior is exemplified by the two-dimensional tracks of Fig. 54-5, where $J_1 = \sin wt$ and $J_2 = \cos wt$ and $S = \sqrt{C_1^2 + C_2^2}$. In all the examples except the first, the jitter amplitude is reduced as the minimum is attained.

In actual practice S is not a function of $C_i$ directly but of their (immediate) pasts. Furthermore, S has only a limited bandwidth (channel capacity), which limits the rate of improvement. We are examining these points further in order to estimate rates of improvement when the number of variable controls is very large.

B. Howland
M. Minsky
O. G. Selfridge

## IV. MISCELLANEOUS

### A. Short Pulse Generator Using Hole Storage Effect

Figure 54-6 shows a diode pulse generator circuit of a sort previously described.* When this particular circuit was tested it unexpectedly produced a pulse power output many times

*Division 3 Quarterly Progress Report, Lincoln Laboratory, M.I.T. (15 January 1958), p. 46, Fig. 34-7.

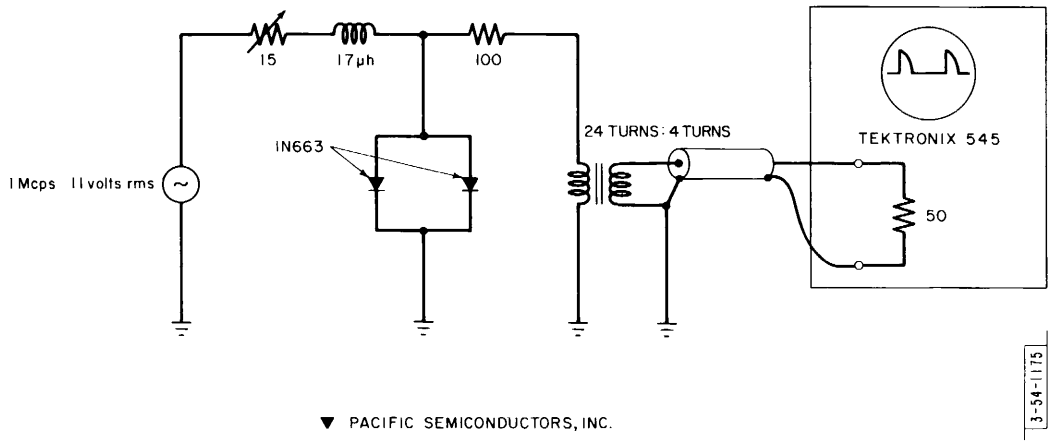


Fig. 54-6. Diode pulse generator.

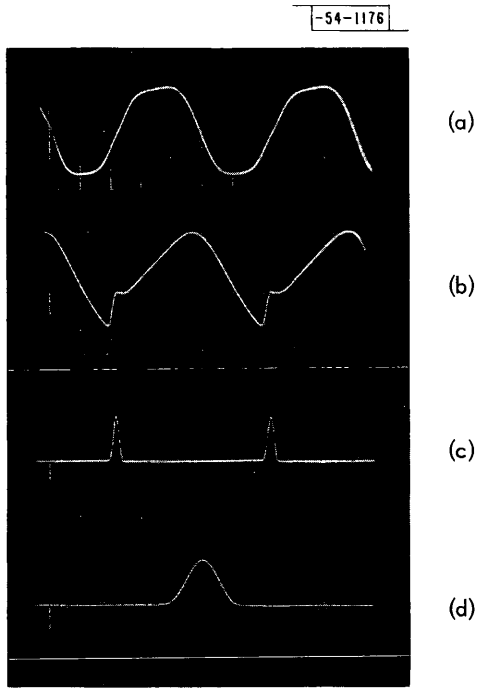


Fig. 54-7. Short pulse generator circuit waveforms.

that which was anticipated. This fortuitous behavior is due to the hole storage effect in the type 1N663 diodes (Pacific Semiconductors, Incorporated), which, at 1 Mcps, permit an appreciable backward current to flow until the supply of holes is exhausted. The energy stored in the inductance coil is, at this instant, released in the form of a short current pulse, which is matched to the low impedance output by a step-down transformer.

Figure 54-7 shows, from top to bottom, the relevant waveforms for this circuit, operating at 1 Mcps: (a) is the input "sine wave" signal; the vertical scale is 10 volts/cm and the circuit load impedance is approximately 200 ohms. (b) is the current waveform through the diode; the vertical scale is 100 ma/cm. The center line here was adjusted to correspond accurately to zero current through the diode; the reversal of current through the diode is apparent. (c) is the pulse output from the circuit into 50 ohms; the vertical scale was 5 volts/cm. (d) is the same pulse with the horizontal scale expanded five times; the sweep speed was 40 μsec/cm.

A possible application of such a circuit is the generation of clock pulses for high speed computers. A contemporary requirement would be 10 watts at a 10-Mcps pulse rate; pulse widths of 10 mμsec would be more than sufficiently narrow for this purpose. These specifications represent an order of magnitude improvement over the circuit of Fig. 54-6 in two directions, and are our immediate goal. Further development is being undertaken in cooperation with Mr. Philip Fleck of Group 34.

B. Howland

## B. Statistics

Let the stochastic process $x(t)$ be the solution of the diffusion equation $dx(t) = m(t, x(t))dt + \sigma(t, x(t))\, dy(t)$ where $y$ is Brownian motion. Let $M$ and $M_\alpha$ be the measures imposed on the space of sample functions by the processes $x(t)$, and $x(t) + \alpha f(t)$, respectively. Then under reasonable assumptions on $m$, $\sigma$, and $f$ the likelihood ratios are determined by a differential equation

$$\frac{d}{d\alpha}\frac{dM_\alpha}{dM}(x) = \varphi(x - \alpha f)\frac{dM_\alpha}{dM}$$

for some $\varphi$. If $\sigma$ is a function of the first variable only,

$$\varphi = \int \frac{f(t)\, m_2(t, x(t)) - f'(t)}{\sigma(t)}\, dy(t) \quad .$$

T.S. Pitcher

Unclassified

# ANALYSIS
# GROUP 55

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ZIERLER, N.

Unclassified

# ANALYSIS
# GROUP 55

## I. INTRODUCTION

Most of the work of Group 55 for the preceding three months has been in the following areas: multiple-station queuing theory, the asymptotic behavior of solutions of certain integro-differential systems, mathematical foundations of quantum mechanics and statistical theory of radar reception. Preliminary results have been obtained in the multiple-station queuing theory which are reported below. This is a subject in which there are a great many unsolved problems (many of which are probably too complex for analytical solution), and work on it is continuing. An integro-differential system describing the dynamic behavior of a continuous medium nuclear reactor has been investigated extensively by one of our group in cooperation with Professor John A. Nohel of the Mathematics Research Center, U.S. Army, University of Wisconsin, and the Georgia Institute of Technology; this work is described in the following paragraphs. Some remaining problems concerning this integro-differential system are being pursued further. A doctoral thesis* on foundations of quantum mechanics written by N. Zierler of this group has been accepted by the Mathematics Department of Harvard University. Some of his more recent results are stated below. A statistical study of radar reception when there are interfering target echoes, or what is sometimes called the problem of radar resolution, is in process by W. L. Root, using techniques developed in TR-158 and TR-159.

W. L. Root

## II. A SYSTEM OF INTEGRO-DIFFERENTIAL EQUATIONS ARISING IN REACTOR DYNAMICS

### A. Introduction

In this report we study existence, uniqueness, and asymptotic behavior (as $t \to \infty$) of the solution of the real system

$$\frac{du}{dt} = -\int_{-\infty}^{\infty} \alpha(x)\, T(x,t)\, dx \quad , \quad a\frac{\partial T}{\partial t} = b\frac{\partial^2 T}{\partial x^2} + \eta(x)\, u \quad (0 \leqslant t < \infty) \quad , \tag{1}$$

satisfying the initial condition

$$u(o) = u_o \quad , \quad T(x,o) = f(x) \quad (-\infty < x < \infty) \quad . \tag{2}$$

In (1) and (2) $u = u(t)$, $T = T(x,t)$ are the unknown functions, $\alpha, \eta, f$ are given real-valued functions, $u_o$ is a given real constant, and $a, b$ are given positive constants.

Of particular interest also is the "degenerate" system (obtained from (1) by putting $b = o$)

$$\frac{du}{dt} = -\int_{-\infty}^{\infty} \alpha(x)\, T(x,t)\, dx \quad , \quad a\frac{\partial T}{\partial t} = \eta(x)\, u \quad , \tag{3}$$

where $u, T$ satisfy (1,2) and where $\alpha, \eta, a$ are the same as in (1). We investigate the relationship of the solution of (1, 2) to that of (3, 2) as $b \to 0+$.

The system (1) describes the linearized dynamic model of a continuous-medium nuclear reactor about its equilibrium state. The interpretation of the various quantities in (1) is

* N. Zierler, "On General Measure Theory," Group Report 55-7 (not generally available).

$t$ = time,

$x$ = position along the reactor (regarded as a doubly infinite rod),

$u(t)$ = logarithm of the total reactor power ($u(t) \equiv 0$ at equilibrium),

$T(x, t)$ = deviation of the temperature from equilibrium ($T(x, t) \equiv 0$ at equilibrium),

$-\alpha(x)$ = ratio of the temperature coefficient of reactivity to the mean life of neutrons,

$y(x)$ = fraction of power generated at $x$,

$a$ = heat capacity,

$b$ = thermal conductivity.

For further physical statements about the dynamics of reactors we refer to Ergen.* An heuristic discussion of the nonlinear problem associated with (1) is contained in the work of Ergen and Nohel.† A rigorous discussion of the stability of a heterogeneous reactor of $n$ different media (the continuous medium reactor may be regarded as a limiting case of such a reactor) is contained in a recent paper by Ergen, Lipkin and Nohel.‡

## B. Summary of Results

The theorems given below are proved in the paper under different hypotheses. For simplicity, we state here only the severest hypotheses, i.e., those under which all of the theorems are true.

Hypothesis

(1) $\alpha, \eta, f$ continuous on $-\infty < x < \infty$

(2) $\alpha(x), \eta(x), f(x) = 0(e^{-\lambda|x|})$ as $|x| \to \infty$ for some $\lambda > 0$

(3) $\mathrm{Re}[\hat{\eta}(x)\,\hat{\alpha}(-x)] \geqslant 0$ , $\mathrm{Re}[\hat{\eta}(x)\,\hat{\alpha}(-x)] \not\equiv 0$ $(0 \leqslant x < \infty)$

$$\left[\text{where} \quad \hat{\eta}(x) = \int_{-\infty}^{\infty} e^{-ixy}\,\eta(y)\,dy\right]$$

(4) $$\left(\int_{-\infty}^{\infty} \alpha(x)\,dx\right)\left(\int_{-\infty}^{\infty} \eta(x)\,dx\right) \neq 0$$

Theorem 1 (Existence). There exists a solution, $u(t; b)$, $T(x, t; b)$ of (1, 2) in the region $-\infty < x < \infty$, $0 \leqslant t < \infty$.

Theorem 2 (Asymptotic Behavior). The solution $u(t; b)$, $T(x, t; b)$ of Theorem 1 has the following behavior. There exists constants $d_i = d_i(b)$ $(i = 1, 2, \ldots)$ such that

$$\lim_{t\to\infty} t^n \left[u(t; b) - \frac{1}{\pi\sqrt{t}} \sum_{j=1}^{n} (-1)^j d_j \frac{\Gamma(j + \frac{1}{2})}{t^j}\right] = 0 \quad (n = 1, 2, \ldots) \quad ,$$

$$T(x, t; b) = 0\left(\frac{1}{\sqrt{t}}\right) \quad (t \to \infty \quad ; \quad \text{uniformly in } x) \quad .$$

The constants $d_i$ are given by specific formulae in terms of $d, \eta, f, u_0$, which are too complicated to give here.

*W.K. Ergen, J. Appl. Phys. 15, 702 (1954).

†W.K. Ergen and J.A. Nohel, J. Nucl. Energy, Part A: Reactor Science 10, 14 (1959).

‡W.K. Ergen, H.J. Lipkin and J.A. Nohel, J. Math Phys. 36, 36 (1957).

Theorem 3 (Uniqueness). There is at most one solution, u(t), T(x, t), of (1, 2) such that

$$\int_0^\infty |u(t)|\, dt < \infty \quad , \quad |T(x,t)| \leqslant K \quad (-\infty < x < \infty \quad , \quad 0 \leqslant t < \infty) \quad ,$$

where K is a constant.

Theorem 4 (Degenerate Case). There is a unique solution, v(t), $\Pi(x, t)$ of (3, 2) and it is given by

$$v(t) = -\frac{(\alpha, f)}{[\frac{1}{a}(\alpha, f)]^{\frac{1}{2}}} \sin[\tfrac{1}{a}(\alpha, f)]^{\frac{1}{2}} t + u_o \cos[\tfrac{1}{a}(\alpha, \eta)]^{\frac{1}{2}} t$$

$$\Pi(x,t) = f(x) + \frac{\eta(x)}{a}\left\{\frac{(\alpha, f)}{\frac{1}{a}(\alpha, \eta)}\left[\cos[\tfrac{1}{a}(\alpha, f)]^{\frac{1}{2}} t - 1\right] + \frac{u_o}{[\frac{1}{a}(\alpha, \eta)]^{\frac{1}{2}}} \sin[\tfrac{1}{a}(\alpha, \eta)]^{\frac{1}{2}} t\right\} \quad .$$

Theorem 5 (Behavior as $b \to 0+$). Let u(t; b), T(x, t; b) and v(t), $\Pi(x, t)$ be as in Theorems 1 and 4 respectively. Let $T_1 > 0$. Then

$$\lim_{b \to 0} u(t; b) = v(t) \quad \text{uniformly over } 0 \leqslant t \leqslant T_1$$

and for every x

$$\lim_{b \to 0} T(x, t; b) = \Pi(x, t) \quad \text{uniformly over } 0 \leqslant t \leqslant T_1 \quad .$$

From Theorem 2 one has

$$\lim_{t \to \infty} t^{3/2}\, u(t; b) = -\frac{1}{2\sqrt{\pi}}\, d_1$$

so that, in particular, u(t; b) is of constant sign for arbitrarily large t. It is interesting to compare this with the periodic nature of v(t) and Theorem 5.

### C. Methods

By a formal calculation the following Volterra equation is obtained

$$u(t) = u_o - \frac{1}{\pi a}\int_0^t \left\{\frac{1}{(x^2 b)/a}\left[1 - \exp\left(-\frac{x^2 b}{a}(t-\tau)\right)\right] h_1(x)\, dx\right\} u(\tau)\, d\tau$$

$$- \frac{1}{\pi}\int_0^\infty \frac{1}{(x^2 b)/a}\left[1 - \exp\left(-\frac{x^2 b}{a}(t-\tau)\right)\right] h_2(x)\, dx$$

where

$$h_1(x) = \mathrm{Re}[\hat{\eta}(x)\, \hat{\alpha}(-x)] \quad , \quad h_2(x) = \mathrm{Re}[\hat{f}(x)\, \hat{\alpha}(-x)] \quad .$$

This Volterra equation is then studied in great detail with the aid of Fourier, Laplace, and Stieltjes transforms; the principal tool being a Tauberian theorem for Laplace transforms. The (unique) solution of this equation is the function u(t; b) above. The function T(x, t; b) is defined, in terms

of u(t; b), by means of well-known formulae in the theory of the heat equation. Various techniques used in ordinary differential equations are also employed.

J. J. Levin
J. A. Nohel*

## III. ON GENERAL MEASURE THEORY

A few final results on the partial ordering of the observables for general measure domains or "event lattices" were obtained (Division 5 Quarterly Progress Report for 15 March 1959). They are described in detail in Section 4 of Group Report 55-7, "On General Measure Theory" (not generally available) and have, as a consequence, a generalization of a theorem of Kadison to the effect that if the set of observables is an anti-lattice in the sense that two of its elements have a sup or inf only when they are comparable, then the center of the event lattice is trivial. The axioms for elementary quantum mechanics have undergone a slight further refinement.

N. Zierler

## IV. NETWORKS AND QUEUES

In the past quarter we have been examining a system of stations $[S_j]$, $j = 1 \ldots M$, where the processing of messages takes place and transmission occurs between stations. In each unit time, $s$ messages are processed at station $S_j$, with probability $q(j, s)$ if more than $s$ messages are present, with probability $\sum_{t=s}^{\infty} q(j, t)$ if exactly $s$ messages are present, and with probability 0 if less than $s$ messages are present; $\sum_{s=0}^{\infty} q(j, s) = 1$. A message processed at station $S_j$ is sent immediately to station $S_k$ with probability $p(j, k)$ regardless of what else may happen at this station or elsewhere; $\sum_{k} p(j, k) = 1$.

The objective is to find expressions for the distribution of busy times at the various stations or sets of stations, and the distribution of maxima during busy times and similar other distributions as a basis for determining the effects of different levels of storage facilities.

One approach to the problem is by the use of generating functions. We let

$$S(j, n, z) = \sum_{s=0}^{n-1} q(j, s)\, z^s + z^n \sum_{s=n}^{\infty} q(j, s) \quad .$$

Since the coefficients of $\prod_{1}^{M} z_i^{t_i}$ in $(\Sigma p(j, k)\, z_k)^t$ give the probability of having $t_1, \ldots, t_M$ messages going to stations $S_1, \ldots, S_M$ respectively if $t$ messages were sent from station $S_j$, the generating function of the distribution of destination of $n_j$ messages sent from station $S_j$ will be given by

$$T_j(\vec{z}, n_j) = z_j^{n_j} S\left[j, n_j \quad , \quad \left(\sum_{k=1}^{M} p(j, k)\, z_k / z_j\right)\right] \quad . \qquad (1)$$

Since each station operates independently of the others, and indicating by $\vec{n} = (n_1, \ldots, n_M)$ the

* Mathematics Research Center, U.S. Army, Madison, Wisconsin and Georgia Institute of Technology. Supported by U.S. Army under Contract No. DA-11-022-ORD-2059, and, in part, by the National Science Foundation, NSF G-7343.

number of messages at the sundry stations at the beginning of a time interval, by $\vec{n}^* = (n_1^*, \ldots, n_M^*)$ the number of messages at the end of the interval, using $\vec{z} = (z_1, \ldots, z_M)$ and $\vec{z}^{\vec{n}}$ to indicate $\prod_{i=1}^{M} z_i^{n_i}$; the generating function

$$G(\vec{z} \mid \vec{n}) = \sum_{\vec{n}^*} p(\vec{n}^* \mid n)\, \vec{z}^{n^*} = \prod_{j=1}^{M} T_j(\vec{z}, n_j)$$

gives the transition probabilities for each time interval.

J. R. Kinney

Unclassified

# PSYCHOLOGY
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# PSYCHOLOGY
# GROUP 58

## I. INTRODUCTION

One of the most persistent problems in the area of man-machine integration concerns procedures for communicating with modern high-speed computers. Present-day procedures require the man to communicate with the machine in special languages that are much closer to the basic computer processes than they are to the man's own capabilities. A more satisfactory state of affairs would exist if the computer could be programmed so that the man could communicate with the machine in his own language. An interlocking set of studies is under way aimed toward the development of procedures for machine recognition of spoken language. Considerable progress has been made on a computer program for separating continuous speech into syllable-like units. A program for differentiating among the vowel sounds of English monosyllables as spoken by many different voices has been improved during the past quarter. Work has begun on a parallel study to test the ability of people to recognize the same vowels. This work will permit comparison of the accuracy of the computer program with that of human listeners. Preliminary results indicate that the computer does very nearly as well as the humans.

A somewhat more immediate problem concerns the use of computers for study of nonnumerical kinds of processes. The coding of the Newell-Shaw-Simon Information Processing Language for the IBM 709 computer will facilitate construction of programs for computer simulation of human cognitive processes, and for study of other nonnumerical symbol manipulation processes.

Work continues on the study of the psychological processes involved in visual pattern perception, quantitative judgment and rote learning. Instrumentation is complete and data gathering has begun for the study of the dark adaptation process of the human eye. In addition, work has begun on a study concerned with the prediction of choices made in a complex judgment situation from subjective ratings of component parts.

A methodological study of the reliability of queuing data has been completed. An equation relating reliability to sample size and load is presented for the type of queue studied.

Group 58 continues to take part in projects outside the group. W. P. Harris' work on aspects of antisubmarine warfare will be reported in a forthcoming Division 2 quarterly progress report. D. B. Yntema has been devoting a considerable portion of his time to the automatic data processing study being carried out for CONARC.

## II. MAN-MACHINE COMMUNICATION

### A. Machine Recognition of Speech

Progress toward the goal of the automatic recognition of human speech continues to be encouraging. During the present reporting period work has continued on the two major aspects of the problem: recognizing isolated monosyllables, and segmenting continuous speech into syllables. In addition, studies have been started to compare computer performance with human performance, using the same speech signals. These comparison studies will be continued as an integral part of the project, to provide a base line for evaluating the automatic recognition scheme.

1. Vowel Recognition

As previously reported (March 1959), a spectrum analysis program for the Whirlwind I computer was developed which classified each 6-msec vowel sample as one of ten vowels and, at the end of the word, chose the vowel with the highest vote as the final classification. The over-all recognition score, as reported, was 88 per cent. The scheduled shutdown of Whirlwind I inhibited the further development of this vowel-recognition program and caused the majority of the effort of this reporting period to be devoted to the analysis of the results already obtained.

The errors made by the recognition program were examined in some detail. They fell into two groups: (a) those in which the majority rule failed because the vowel was changing with time, and (b) those which were outright failures of the formant-location and confusion-elimination operations to correctly identify a steady spectrum. Nineteen of the 27 errors were of the first type. These were approximately evenly divided among: (a) errors due to /b/ transitions, (b) errors due to /t/ transitions, and (c) errors due to words pronounced as diphthongs. A brief study of the remaining eight errors indicated that seven of them could probably be corrected by small changes to the present program (e.g., making an absolute threshold depend upon pitch).

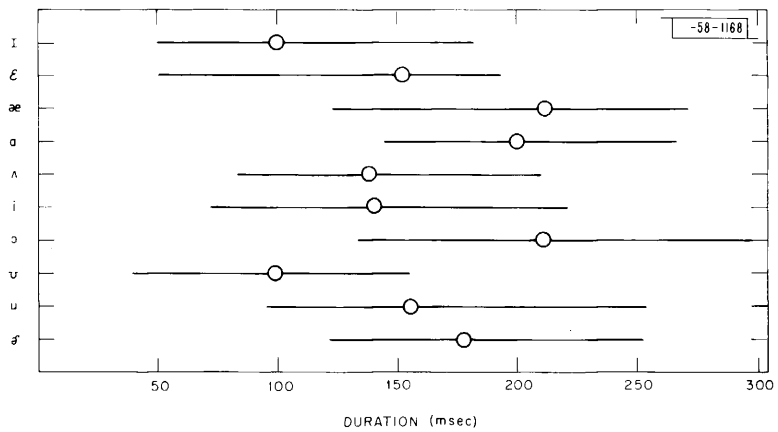


Fig. 58-1. The range of the duration of each vowel is shown with the average indicated by a circle.

The use of duration information to improve the recognition score was investigated. Figure 58-1 shows the averages and extremes of duration for all words used in these experiments. A simple rule was adopted to utilize this information. When the program had completed its plurality-rule classification, the duration of the unknown vowel was checked against the range of durations for the vowel having the highest vote. If the range included the range of the vowel in question, the vote was accepted as correct. If not, the duration of the second highest vowel was checked, etc., until a compatible range was found. With the application of this rule, the percentage correct identification rose to 93.

A paper on this work was written and is being submitted to the Journal of the Acoustical Society of America for publication.

A brief study was made of the effect on the recognition score of adding random noise in the band 100 to 10,000 cps to the speech input. On the words of the one speaker used, the program scored 100 per cent without noise. With the noise added at a 0-db signal-to-noise ratio (peak

signal power equaled average noise power), the recognition score was still over 50 per cent. At −6 db, it had fallen to just over 25 per cent.

To study the effects of different consonant environments, the recognition program was run on 100 words with various initial and final consonants from each of two speakers (1 male, 1 female). The score dropped to around 60 to 65 per cent. The reasons for some errors are known and corrections planned. The reasons for others are being studied.

The conclusions reached from the analysis of the results of the Whirlwind I program are now being incorporated into the first TX-2 speech-recognition program.

Carma D. Forgie
J. W. Forgie (Group 51)

2. Intelligibility Testing

A study is being carried out to compare the responses generated by the Whirlwind I program for vowel recognition with responses generated by human listeners on the same material. The test involves the ability to recognize the vowels represented by the monosyllables bit, bet, bot, bat, but, beet, boot, bought, put, bert, and book. The stimuli for the human listeners, taken from the tape recording used in evaluating the computer program, consist of a randomly arranged list of 215 monosyllables, spoken by 21 different people.

Preliminary results show that some of the stimuli in the recording are consistently confused with monosyllables other than those that were intended to have been spoken. For example, word number 165 in the listening test was intended to be "bat," but has been responded to as "bet" by both the computer and eight of the nine listeners tested.

Recognition scores for the nine listeners tested thus far range from 95 to 99 per cent as compared with 93 per cent for the computer program. More data are now being collected.

M. L. Groves

3. Speech Segmentation

Work continues on development of a method for segmenting speech into syllable-like units. The over-all system, according to present plan, is diagrammed in Fig. 58-2. Eight detectors are used to detect features or configurations of the amplitude envelope of the speech wave. The output lines of these detectors are the inputs to a logical decision network that determines, on the basis of the occurrence of the features in succession and in combination, the points in time at which boundaries of syllable-like units occur. The features being used at present are: (a) high amplitude, used as an aid in recognizing the occurrence of vowels; (b) hiatus, used as a cue to recognize the occurrence of stop consonants; (c) long hiatus, or a stretch of silence longer than that which may occur within a syllable; (d) voicing, used to recognize a potential syllable nucleus, and used to recognize structurally significant combinations of voiced and unvoiced phonemes; (e) negative-to-positive slope change: a boundary feature associated with intervocalic fricatives, nasals, and semivowels in certain cases; (f) fast slope change, used to recognize a boundary point in certain successions of vowels; (g) voiced peak, used to recognize a syllable nucleus of low amplitude; (h) stop-consonant-type peak, used to recognize the occurrence of aspirated stops.

The system is being simulated on the IBM 709. At present programs have been written and checked out which simulate: (a) the array of detectors shown in Fig. 58-2. This program also displays the envelope on the CRT. An x-axis is drawn at the points where the particular features are detected. (b) The states, either "on" or "off," of the output lines of the detectors. (c) The

logical decision network. The output of this program may be used in combination with (a) in order to display the envelope on the CRT with an axis drawn at the points where syllable boundaries occur.

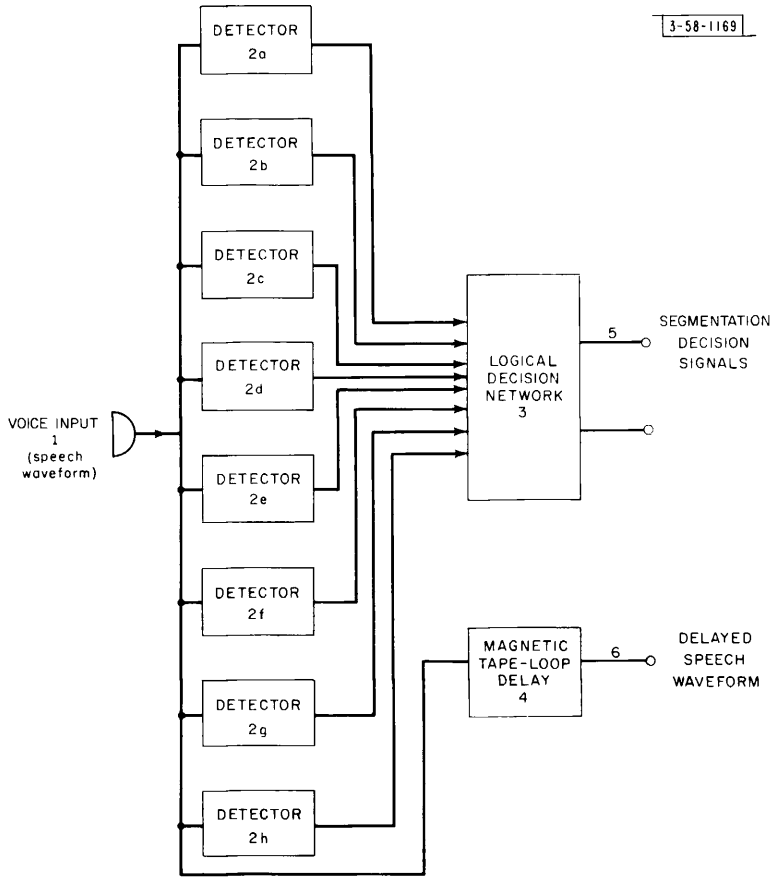


Fig. 58-2. Over-all plan of segmentation device.

In addition a program has been written and checked out which synchronizes the syllable boundary points with the original speech wave, so that one may listen to the individual syllables.

A. N. Stowe

## B. Information Processing Language for the IBM 709 Computer

Most present-day digital computers are designed primarily for numerical computation, and the programming languages that have been developed to facilitate computer programming are likewise oriented toward computation. It is relatively difficult to write digital computer programs for complex symbol-manipulation processes of a nonnumerical type. The difficulty arises, for example, in writing programs that would allow the computer to answer questions posed in English about a file of data in its storage. Chess playing, theorem proving, and the computer simulation of human cognitive processes involved in problem-solving and decision-making are also difficult to program in the standard computer languages.

Newell, Simon, Shaw and their collaborators have designed a special information processing language (IPL) for programming complex information processes. The main feature of the language is the organization of computer storage in lists and list structures. A list of symbols consists of a set of memory registers each holding the given symbol, and also containing the address

of the next register on the list. Each symbol can be found in turn by starting at the head of the list, since each cell is linked to the next. This memory structure is very flexible. For example, it permits a symbol to be inserted on a list by just putting the symbol in a cell and adjusting the linkages. Also the list need not be stored in successive locations in storage. A symbol on a list can be the name or address of another list. A list structure is a hierarchy, including a main list, all lists named on the main list, all lists named on these lists, etc. The IPL system includes a large number of processes for manipulating lists and list structures.

An IPL system has been coded for the Laboratory's IBM 709 computer. The system includes an interpreter and a set of processes so that programs written in IPL-V, the latest version of the language, can be executed on the 709.* Similar IPL systems have been coded elsewhere for the IBM 704 and 650, and are compatible with the 709 system. It will thus be possible to exchange programs with workers at other laboratories. The 709 version of the IPL system has several additional features not currently available in the 704 and 650 systems. Postmortem and snapshot programs have been incorporated to facilitate debugging. These, as well as the trace program available in all IPL systems, have the advantage of communicating with the programmer in the symbols he used in writing his program, rather than in the numbers used by the computer. The trace routine records the current status of relevant cells in the system as each instruction is executed, the snapshot records the contents of all common storage lists and the postmortem routine records all routine and data lists in the computer's storage.

B. F. Green, Jr.
Alice K. Wolf

## III. HUMAN INFORMATION PROCESSING

### A. Tracking Targets Through Noise

The ability of human operators to track targets on visually noisy displays has been studied in previous experiments, in which the displays were shown to the operators at a rate of one display every 3 seconds. During this reporting period, a preliminary experiment has been completed to evaluate the utility of a motion picture presentation, with a rate of 20 displays per second.

The displays used were computer-generated film strips, each containing one, two, or four targets that started outside a field of randomly positioned noise dots, moved on a straight course into the noise, and turned once at a randomly determined point. Each film strip was repeated from beginning to end twelve times before the film projector was stopped. The operator's task was to mark the last position in each target's course as quickly as possible. Both the speed and accuracy of his performance were recorded. The experimental variables were the number of targets to be tracked, the noise level, the targets' blip-scan ratio, and the over-all size of the projected display.

In this preliminary experiment, the number of trials per condition was very small, so that the data obtained are not conclusive. However, the data do suggest that the technique of presenting the display at a rapid rate enables operators to track targets, in some noise and blip-scan conditions, much faster than they can when the displays are presented at a slow, conventional rate. Furthermore, it appears that, for some conditions, tracking accuracy will be better than that obtained at the slow rate. For example, with four targets, 0.75 blip scan, and high noise level, the

* A manual (Lincoln Laboratory Group Report 58-4 – not generally available) describing the use of IPL-V, the latest version of the language, has been reproduced and distributed.

average number of targets correctly tracked in the present experiment was 1.75 as compared with 0.84 for the same conditions with a slow display rate. It appears that a small increase in the number of repetitions of the film strip would further improve performance, while maintaining a considerable time advantage over the slow display rate.

Alice K. Wolf
B. F. Green, Jr.

### B. Kinetic Depth Effect

Previous experiments (reported in the Division 3 Quarterly Progress Report dated 15 April 1958) with the ability of human operators to utilize information about the third dimension from a moving two-dimensional display have indicated accuracy was achieved only with relatively long exposure times – 30 seconds or more with simple 5-element patterns. These patterns were generated by the shadows cast on a translucent screen by five pegs positioned in a $4 \times 5$ matrix on a rotating turntable. Observers were asked to indicate what matrix positions were occupied by pegs. It is of some interest to compare observer performance on this moving two-dimensional display with performance on the same patterns viewed directly in a static display. The display was modified by removing the translucent screen on which the shadows were cast and providing a brightly illuminated white background against which the pegs were silhouetted. The observers viewed this display with the head immobilized to minimize movement parallax cues. Two conditions of observation were employed: monocular and binocular. Under binocular observation with a 30-second exposure time, accuracy of performance was slightly better than that obtained under the most favorable condition with the moving displays. Accuracy with monocular observation was about as poor as the least favorable condition with the moving displays. Thus, at least under some conditions, human operators can make as good use of a static display that employs binocular parallax as they can of a display that employs motion to generate depth.

B. W. White
Gayle E. Mueser

### C. Pattern Recognition Display Program

One series of 50 patterns generated by the display program described in the Division 3 Quarterly Progress Report dated 15 October 1958 was used in a study at the Ohio State University to determine how accurately human observers could tell which of two adjacent patterns in a series was more random. Step size in the random walk was constant throughout this series. The discrimination was found to increase markedly in difficulty as the series progressed, the judgments falling to chance expectancy about midway in the series. The program has now been modified to permit a change in step size with each frame, with the hope that a series can now be generated which exhibits equal discriminability throughout.

B. W. White

### D. Prediction of Complex Judgments

Large man-computer systems are often required to make important decisions that are likely to involve factors that cannot be foreseen, or cannot, in practice, be considered in the computer program. In such cases it is not usually considered safe to let the machine make the decision by itself; human judgment is regarded as essential. How to use a man efficiently in such situations is a perennial problem of system design. The man may be required to take action on each situation, or he may monitor the machine's actions and stand by to intervene when necessary. But in

any event, he is usually required to consider all difficult situations one by one and decide what action is required. Often he finds himself making essentially the same decision over and over, applying repeatedly a temporary set of decision rules that takes account of the special factors that the computer is not programmed to consider. If there were an easy way for him to tell the machine what these rules were, then he could be spared the labor of examining each situation, and the machine could proceed to make decisions at computer speed until he changed his instructions.

One possible technique is being investigated. The experimental setting concerns the attractiveness of a number of hypothetical jobs, which are described in terms of three attributes: salary, quality of co-workers, and permanence. The problem is to determine whether a subject's evaluation of the importance of each separate attribute and the spacing of steps within each attribute, can be combined in a simple additive manner to predict his evaluation of the attractiveness of each total job. The results have been encouraging enough so that this way of allowing a person to summarize his judgments will be investigated further.

D. B. Yntema
W. S. Torgerson
Laura Klem

### E. Quantitative Judgment

An experiment comparing ratio and distance judgments of both lightness and darkness of Munsell color chips was repeated using a new sample of subjects. The results confirm those obtained earlier: scales derived from ratio judgments indicate that subjective lightness is the reciprocal of subjective darkness, whereas, for scales derived from distance judgments, subjective lightness is the negative of subjective darkness. For both lightness and darkness, the "distance" scale is approximately the logarithm of the "ratio" scale. The results are consistent with the notion that the subject perceives a single type of quantitative relation between stimuli, which he interprets as a ratio relation in one case and as a distance relation in the other.

W. S. Torgerson
Gayle E. Mueser

### F. Visual Psychophysics

Instrumentation for the investigation of dark adaptation of the human eye described in the last quarterly progress report (Division 5 Quarterly Progress Report for 15 March 1959) is virtually complete. Construction of the threshold measuring equipment and the unit for controlling light adaptation is finished. Calibration of these units and their associated filters and wedges is 95 per cent completed. Some preliminary practice runs have been made with three observers. Comparison of threshold data for two of these observers with data taken earlier on another instrument shows agreement within about 1 db. This gratifying correspondence gives us considerable confidence in the validity of the calibrations. Serious data taking should begin the first week in June.

R. T. Mitchell
Rhoda Resnick

### G. Rote Verbal Learning

Final arrangements have been completed for carrying out an experiment, discussed in the preceding quarterly progress report, on the learning of monosyllables under various conditions

of exposure and recall of the items. This experiment is to begin as soon as subjects, who will be provided by the M.I.T. Student Placement Bureau, become available.

An experiment to determine how many monosyllables, in lists of different lengths, are recalled after one exposure of each list is at present under way.

Nancy C. Waugh

## IV. METHODOLOGY

### A. Reliability of Queuing Data

The study of the reliability of queuing data discussed in the preceding quarterly progress report has now been completed. The queue studied was the one-server, random-input, constant-service time system. The particular measure of reliability used was the maximum difference between the waiting time distribution of a sample, with load $\Delta$ and sample size N, and the theoretical distribution. This measure was shown to vary inversely with the square root of N and directly with $\Delta$ according to the equation

$$d_{q,\Delta} = \frac{\sqrt{\Delta}(XZ_q - Y\Delta)}{(1-\Delta)\sqrt{N}} \quad ,$$

where $d_{q,\Delta}$ is the maximum difference exceeded by q per cent of samples of size N, $\Delta$ is the system load (input rate/service rate), and $Z_q$ is the q per cent point of the Smirnov* distribution of maximum deviations for independent samples. The parameters X and Y were estimated from the data ($\hat{X} = 0.83$, $\hat{Y} = 0.39$).

The demonstrated lack of reliability in pilot studies or Monte Carlo studies of queuing systems is discussed in a forthcoming group report, along with suggestions concerning how this lack may be overcome. It is suggested that such studies should consist of a fair number of independent samples, rather than one large sample, so that reliability measures can be made. It is also suggested that pilot studies may be more usefully employed to estimate system parameters for later use in computer simulation than used directly to estimate system performance.

J. E. K. Smith
Laura Klem

* N. Smirnov, Ann. Math. Stat. 19, 279 (1948).